Non-Precautionary Cash Hoarding and the Evolution of Growth Firms

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Abstract

We analyze whether growth firms should hoard cash (and delay investment) or incur dilution from external financing. Such non-precautionary hoarding features a self-reinforcing effect: firms with better investment opportunities hoard less, yet grow successful and cash-rich more quickly. Furthermore, hoarding interacts with how growth firms choose between public and private financing and how they react to product market competition. Our insights contrast with those from precautionary theories, and can help explain puzzling empirical evidence, such as why firms choosing private over public financing hoard less, and why increased competition pushes some growth firms to public and other to private financing.

Keywords: cash hoarding; cash holdings; public versus private financing; growth firms; competition; real options

JEL Classification: G31, G32, D92

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1 Introduction

Our knowledge about cash hoarding and investment is mainly framed by a literature that seeks to explain empirical patterns in large and mature firms. Some of the most important rationales for cash hoarding identified by this literature include building up cash reserves for precautionary or tax reasons (Opler et al., 1999; Bates et al., 2009). In this paper, we take a somewhat different perspective: that of a growth firm with investment opportunities already present, but without the necessary cash to undertake these opportunities—arguably one of the most important settings in corporate finance. The relevant question for such a growth firm is whether it should hoard cash to reduce dilution associated with costly external financing (by self-financing more), realizing that this would delay investment; or not hoard, accept dilution and invest immediately. Such non-precautionary hoarding shapes the evolution of growth firms in a number of ways. In particular, we are interested in the interaction between hoarding, the choice of public or private financing, and the firm's competitive environment.

Key for this interaction is the insight that there is a stark difference in predictions depending on whether or not investment opportunities are already present. In our non-precautionary setting, hoarding implies delaying investment of existing growth opportunities. Such delay is more costly for firms with better opportunities. Hence, they delay and hoard less. By contrast, the primary focus in the literature is on precautionary hoarding, i.e., *prior* to the arrival of investment opportunities. In that case, firms hoard *more* when expecting better opportunities (Bates et al., 2009).

By highlighting this fundamental contrast, this paper sheds some light on several puzzling stylized facts. One such fact is that firms choosing private over public financing hoard less cash (Gao et al., 2013; Asker et al., 2015). This finding has been difficult to reconcile with the idea that firms hoard cash as a precaution, given that raising a dollar from a private financier might be more expensive due to illiquidity costs. However, we argue that such evidence would emerge naturally when firms already face a funding shortage and delay investment to hoard. We further ask to what extent product market competition affects the public-private choice. The present evidence is contradictory, with some empirical studies showing that competition increases (Chod and Lyandres, 2011), while other showing that it reduces the preference for public financing (Chemmanur et al., 2010). Our paper shows that such opposing findings could, indeed, emerge when firms are cash-constrained and need to think not only about how competition threatens their first-mover advantage, as emphasized by the prior literature (Grenadier, 2002), but also about how it affects their incentives to delay investment to hoard cash.

We derive our insights in a simple dynamic model in which a firm wants to make something "big and bold." External financing is costly, making cash hoarding a natural way for growth businesses to gain some elbow room and reduce dependence on costly external financing. In our baseline model, the cost of finance is due to investors having a more subdued valuation of the firm's growth options (Dittmar and Thakor, 2007; van den Steen, 2005, 2010). However, we generalize our results also to other financing frictions, such as the dilution of effort incentives when partially relinquishing ownership.

The cost of non-precautionary cash hoarding is that it may lead to delays in existing investment opportunities. The basic and robust insight that serves as our starting point is that firms hoard less when their investment opportunities are better. Intuitively, such firms are willing to depend more on external funding, because the cost of investment delay is increasing in the attractiveness of the opportunities. Already this simple insight has novel implications for the role of cash in the evolution of growth firms. It shows that there is a self-reinforcing effect, where firms with a higher growth potential also choose to expand more quickly. Furthermore, being more successful, these firms might end up being cash-rich as they mature despite pursuing a low-cash strategy in their growth phase. Building on these insights we derive a number of novel results regarding how cash hoarding affects financing and investment strategies.

Stark implications come up when relating the interaction of non-precautionary cash hoarding and investment to a firm's choice between public and private financing. A cost of private financing is that firms face financiers that need to be compensated for the cost of holding illiquid claims. A benefit is the closer involvement of financiers, which can lead to a better alignment of how they view the firm's growth opportunities.[1] We show that such alignment is most valuable for firms that choose to depend more on external financing, i.e., firms that want to invest more quickly and hoard less. Thus, in line with the empirical evidence, firms choosing private financing hoard less.

The preceding results hold true despite the fact that private financiers could have stronger bargaining power, which could make their financing costlier. A natural concern is that this could offset the benefit of closer alignment. However, private financiers take into account the firm's outside option of raising public financing and internalize the impact of their bargaining power on cash hoarding and investment delay. Thus, although the lack of competition among private financiers affects the firm's profits, it plays a secondary role for the decision to raise public or private financing and non-precautionary hoarding. This is in contrast to the effect of product market competition, which does not feature such

[1] Note that we are not comparing a typical large public with small private firm, but our focus is on a firm for which the choice between public and private financing is endogenous.

endogenous mitigating effects.

Another novel insight of our model is that product market competition has a dual effect on both hoarding and the firm's choice between public and private financing. On the one hand, our model features the standard incentives to speed up investment in the face of competition based on the threat of losing one's first-mover advantage (Grenadier, 2002). On the other hand, our setting highlights that there is a countervailing force: an increase in competition reduces profits regardless of whether or not the firm is a first-mover, which makes delaying investment to hoard cash less costly. Therefore, the standard prediction that competition will speed up investment and market entry, which would imply less hoarding, will be reversed if having a first-mover advantage is not of paramount importance. Relating these insights to our predictions on public and private financing, we expect that a firm for which competition substantially decreases profits will be more willing to hoard and delay investments. Hence, it will seek (less aligned) public financing. Alternatively, if the first-mover advantage dominates, competition makes delay and hoarding less desirable. In this case, more aligned private financing dominates.

We extend the model along several dimensions. First, we show that the firm's cash hoarding policy can reveal its growth prospects in the presence of asymmetric information. More specifically, firms with better investment opportunities hoard even less to signal quality, which qualitatively leaves our remaining results unchanged. Second, we extend our model to discuss optimal financing and payout policies. Paying out cash would not be optimal as it worsens the funding problem. Furthermore, if firms can issue less-disagreement sensitive securities, such as debt, they hoard less cash. Third, non-precautionary hoarding with the same qualitative implications could alternatively arise when external financing dilutes effort incentives ala Holmstrom and Tirole (1997). We further obtain the same results for the interaction between hoarding and the choice between public and private financing when considering alternative trade-offs, based on differential monitoring. The results are also robust to other extensions, such as allowing for investment delay to reduce uncertainty.

The results of our paper could reconcile a number of puzzling empirical findings and give rise to novel empirical predictions. First, our model can explain the counter-intuitive findings (from a precautionary perspective) that private firms hoard less cash than public firms (Gao et al., 2013; Asker et al., 2015). Second, our results demonstrate how the public-private choice is affected by product market competition. In particular, the importance of having a first-mover advantage determines not only hoarding, but also whether competition leads to more public or more private financing. This could help explain why some studies find that competition increases a firm's propensity to raise public financing

(Chod and Lyandres, 2011), while other studies find the exact opposite (Chemmanur et al., 2010). Indeed, the non-monotonicity between investment delay and competition behind our predictions is consistent with existing empirical findings (Akdogu and MacKay, 2008). Third, our analysis points to a self-reinforcing mechanism, in which firms with better opportunities also invest more quickly. As these firms mature, they are likely to end up with large cash holdings due to their success despite their original low-cash strategy—a cash lifecycle prediction finding support in Drobetz et al. (2015). Other predictions include that firms engaging in non-precautionary hoarding should have higher announcement returns when making new investments with a higher proportion of outside (public) financing.

Our paper mainly relates to the fast growing literature on cash. Firms hoard cash because they may be unable to frictionlessly raise financing for new investments. Agency conflicts are one such important friction (Jensen, 1986).[2] Alternatively, firms may hoard cash as a precautionary measure. Bolton et al. (2011) show that firms will keep a positive cash balance even if this necessitates costly external financing, since the marginal benefit of avoiding to seize operations is very high. In such cases, firms with stronger cash flow streams need to hoard less cash (Acharya et al., 2012). Related, Almeida et al. (2004) show that financially constrained firms save more cash out of cash flows.[3] Existing evidence finds strong support for the precautionary motive for hoarding cash (Opler et al., 1999; Bates et al., 2009). However, we are not aware of empirical work explicitly investigating delay of investment due to cash hoarding. In this paper we argue that this channel is important, as such non-precautionary hoarding has contrasting cross-sectional implications compared to precautionary hoarding. This could help explain puzzling patterns, such as in firms choosing private over public financing.

Our analysis provides novel insights about the endogenous relation between cash hoarding, competition, and the decision to raise public or private financing. This analysis is the key difference to prior work, such as Boyle and Guthrie (2003), Hugonnier et al. (2015), and Bolton et al. (2013), which also analyzes cash hoarding in the context of lumpy investment and costly external financing. In particular, our paper is the first to derive the contrasting predictions of precautionary and non-precautionary hoarding, and to analyze

[2]Dittmar and Mahrt-Smith (2007) and Pinkowitz et al. (2006) show that cash is worth less when agency problems between inside and outside shareholders are greater, and Nikolov and Whited (2013) identify low managerial ownership as a key factor driving agency costs. In contrast, Opler et al. (1999) and Bates et al. (2009) find no evidence relating agency problems to cash holdings.

[3]The precautionary motive is key also in the risk management literature. Acharya et al. (2013) show that firms with high aggregate risk exposure prefer cash to credit lines, and Rampini and Viswanathan (2010) argue that the opportunity cost of risk management is higher for constrained firms. Unlike our focus on financing growth opportunities, these papers focus on cash and/or credit lines as means of overcoming liquidity problems. Also note that credit lines are not common for growth firms (Sufi, 2009).

their implications.

Furthermore, our result that product market competition has a *dual* effect on investment delay and hoarding brings forth new aspects relative to prior contributions, such as Hoberg et al. (2012) and Morellec et al. (2014). In these papers, hoarding does not affect the timing of investment, but insures against negative liquidity shocks in order to secure survival. The latter is more at risk in a competitive environment, leading to an unambiguously positive relationship between hoarding and competition. Instead, by focusing on the timing of investment, we highlight that the pressure of competition on future profits makes investment delay and hoarding more attractive. This is a novel insight. It further shows that there is a force working in the opposite direction to the standard argument that competition creates incentives to invest more quickly (Grenadier, 2002; Novy-Marx, 2007). We show that what determines the overall effect of stronger competition on non-precautionary hoarding is how strongly it erodes profits relative to the benefit of being a first-mover.[4]

Our paper is organized as follows. Section 2 presents the model. Section 3 introduces non-precautionary hoarding and relates it to the choice between private and public financing and the effects of competition. In section 4, we analyze various extensions. Section 5 discusses empirical implications. Section 6 concludes.

2 Model

Our baseline model features a firm run by a sole owner-manager (henceforth, manager). This firm already generates revenues, but is still a growth firm with a potentially profitable expansion still ahead of it. We model this in the following natural way. Suppose that the firm has an existing asset in place producing stochastic cash flows. If they are not paid out or invested, these cash flows accumulate in the form of cash reserves within the firm according to

$$dw_t = \mu w_t dt + \sigma w_t dZ_t,\ w_0 > 0, \tag{1}$$

where $\mu > 0$ and $\sigma \geq 0$ are constant and $(Z_t)_{t\geq 0}$ is a standard Brownian motion. This simple reduced-form formulation for how the level of cash changes within the firm is sufficient for our purposes. A key assumption is that $\mu < r$, where r is the constant discount rate used by all. This assumption, which is standard in the real options literature, implies that the firm has only a weak ability to generate cash, and keeping the firm going is costly

[4]Lyandres and Palazzo (2015) also highlight the effect of deteriorating profits on hoarding, but in their model hoarding is precautionary and is *negatively* impacted by competition.

to insiders. Specifically, in our setting, this assumption implies that the opportunity cost associated with hoarding cash is paying out w_t together with effectively closing operations. This is precisely the feature we want to capture for a growth firm for which the investment opportunity is the main component of valuation and absent which the firm constitutes an unprofitable business. At the same time, this setting is sufficiently flexible to allow us to discuss payout policies and to capture the likelihood of default (because if w_t hits zero, the firm does not recover).[5] Though for most of the main text, we refer to w_t as cash, an alternative interpretation is that w_t represents the net worth the firm builds up over time, which is available as a safe collateral free of any financing frictions.

The reason the manager is willing to keep the firm going is that this firm has generated a profitable investment opportunity, requiring an investment of K. Initially, the manager does not have sufficient cash at hand for making the investment, but she has discretion over the timing of the investment. Our approach makes use of the standard real options framework (McDonald and Siegel, 1986; Dixit and Pindyck, 1994), but differs from this framework in one important aspect: the firm is cash-constrained and the manager may not be able or willing to invest in a positive NPV project even if she can raise capital from financiers in a competitive capital market.[6] In our baseline model, the unwillingness to raise external financing is due to differences in vision leading the entrepreneur to believe that her firm is undervalued by outsiders. However, the idea that growth firms hoard non-precautionary cash generalizes to other financing frictions that make external financing costly (Section 4).

Vision and Disagreement At $t = 0$, the manager and the financier publicly observe a signal about the profitability of the project. If the signal is good and the firm invests K, its discounted expected cash flows become θX_G. If the signal is bad and the firm invests K, its expected discounted cash flows become θX_B. The parameter θ can be seen as a publicly observable indicator of the attractiveness of the firm's (or industry's) growth prospects. We assume that $K > \theta X_B$ for all θ, so that a bad signal translates into a negative NPV project. Furthermore, $\theta X_G > K$ at least for some θ, so that investing after a good signal could increase value. All features of our model are common knowledge, and the cash flows and the level of cash are costlessly verifiable. Furthermore, we assume that all parties are

[5]The main advantage of (1) is that it allows us to solve everything in closed form. At the cost of losing this tractability, we could specify a cash flow process generating the cash level w_t as in Bolton et al. (2013), but such alternative formulations do not lead to further insights.

[6]Related to our finance application is the work of Grenadier and Malenko (2011) who analyze real options signaling games. Morellec and Schürhoff (2011) and Bouvard (2014) analyze financial contracting and real options financing under asymmetric information.

risk neutral and protected by limited liability.

The main feature of our vision- (or disagreement-) based model is that although both parties observe the same signal, they may interpret it differently. More specifically, whatever inference is made by the manager, the financier believes that the inference is correct only with probability $\rho \in (0,1)$. The agreement parameter ρ is common knowledge and might depend on the nature of the investment opportunity.[7] In our baseline model, ρ and the expected discounted cash flows prior to investment θX are initially constant over time.

In what follows, we only need to consider the case in which the manager observes a good signal (i.e., infers that the project is good). This is because if the manager observes a bad signal and the financier agrees that the project is bad, it is never undertaken and the cash at hand is paid out. If the financier disagrees and believes that the project is good, the best course of action for the manager is to pay out all available cash and sell the firm to the external financier. Hence, when the manager receives a bad signal, there will not be any hoarding, and the cash at hand is paid out regardless of whether the financier agrees or disagrees.[8]

Outside Financing We initially assume that at the time at which the manager raises capital to make the investment, she is facing a competitive capital market. We model this by allowing the manager to make a take-it-or-leave-it offer for which the financier just breaks even.[9] The manager sells equity to raise $K - w$ for the investment outlay K. The financier's equity stake α must satisfy

$$\begin{aligned} K - w &= \alpha\theta\left(\rho X_G + (1-\rho) X_B\right) \qquad (2) \\ &= \alpha\theta W^F \end{aligned}$$

[7] Disagreement in a corporate finance context is usually introduced by postulating heterogeneous priors in the sense of Kurz (1994a,b)–e.g., Boot et al. (2008). However, disagreement can also arise due to overconfidence on the part of either management or financiers (Bernardo and Welch, 2001; Daniel et al., 1998), and alternatively through excessive pessimism (Coval and Thakor, 1998) or optimism (Manove and Padilla, 1999).

[8] The reduced-form framework we have presented above can be generalized along a number of dimensions. First, it easily extends to introducing a general process for the cash flows generated after investment. Second, we can make the NPV of the investment opportunity stochastic, and we could allow for delay to help resolve disagreement. Finally, instead of disagreement, we could assume that the manager's reluctance to raise external financing is due to the negative effect this would have on her incentives to exert effort (as in Holmstrom and Tirole, 1997). We discuss these and other extensions, such as introducing information asymmetry, in Section 4.

[9] To streamline the exposition, we add more structure to the analysis in the following sections. In particular, we analyze the choice between public and private financing and allow for bargaining power to be in the hands of financiers in the case of private financing.

where $W^F := \rho X_G + (1-\rho) X_B$. θW^F stands for the financier's assessment of the firm's value, capturing that the financier shares the manager's assessment with probability ρ, and disagrees with probability $(1-\rho)$. From (2), the equity share that needs to be promised to the financier is

$$\alpha = \frac{K-w}{\theta W^F}. \tag{3}$$

Define $\theta W^M := \theta X_G$ as the manager's assessment of the firm's value. The manager's net expected payoff at the point in time that she raises $K - w_t$ and co-invests w_t is:

$$V(w_t, \theta) := \left(1 - \frac{K-w_t}{\theta W^F}\right) \theta W^M - w_t. \tag{4}$$

This payoff is increasing in the co-investment w_t, the profitability parameter θ, and the agreement parameter ρ.

3 The Growth Firm's Decision to Hoard Cash

3.1 Non-Precautionary Cash Hoarding and Speed of Growth

We will now derive how the potential lack of alignment between management and financiers—inducing disagreement—affects the delay of investment decisions, and in turn impacts the amount of non-precautionary cash hoarding prior to undertaking the investment. We solve for the value of the real option to invest using standard dynamic programming methods (Dixit and Pindyck, 1994). The problem is that of finding the optimal stopping rule w^* (i.e., the level of cash holdings at which the investment is made) that maximizes the value of the option to invest U. This involves trading-off the benefit of reducing the funding cost against the time value of money lost from investing later, where the manager's expected payoff is

$$U(w_t, w^*, \theta) = \max E\left[\frac{1}{1+rdt}\left[U(w_t + dw_t, w^*, \theta)\right]\right]. \tag{5}$$

Applying Ito's lemma, we obtain

$$rU = \mu w \frac{\partial U}{\partial w} + \frac{1}{2}\sigma^2 w^2 \frac{\partial^2 U}{\partial w^2}.$$

This equation is solved subject to the following boundary conditions. First, the manager's expected payoff at the time of investment should be equal to her payoff from investment: $U(w_t, w^*, \theta)|_{w_t = w^*} = V(w_t, \theta)|_{w_t = w^*}$. Second, the manager chooses the investment trigger so as to maximize her value at the endogenous investment threshold:

$\frac{\partial}{\partial w^*} U(w_t, w^*, \theta)|_{w_t=w^*} = 0$. Finally, the option to hoard cash becomes worthless as the value of cash tends to zero: $\lim_{w_t \to 0} U(w_t, w^*, \theta) = \max\left[0, \left(1 - \frac{K}{\theta W^F}\right)\theta W^M\right]$. Indeed, if the existing business falters ($w_t \to 0$), then it almost surely does not recover (cf. (1)), and the manager can invest only if she raises all financing externally. If that is not possible, the firm has no purpose, and seizes to exist.

Suppose, first, that disagreement is sufficiently strong such that $K > \theta W^F$. Then, solving this optimization problem yields the following expression for the manager's expected payoff

$$U(w_t, w^*, \theta) = \left(\left(1 - \frac{K - w^*}{\theta W^F}\right)\theta W^M - w^*\right)\left(\frac{w_t}{w^*}\right)^\beta \tag{6}$$

where β is the positive root of $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$, and $\mu < r$ implies that $\beta > 1$. Intuitively, the expression in (6) can be interpreted as the manager's expected payoff from investing at w^* multiplied by the probability of reaching the cash level w^* and investing. Trading off the marginal cost (due to $\mu < r$) and the benefit of delay, the value maximizing investment threshold w^* is given by

$$w^* = \min\left[\frac{\beta}{\beta - 1}\left(K - \theta W^F\right)\frac{W^M}{(W^M - W^F)}, K\right], \tag{7}$$

where the upper bound on w^* follows from the restriction that $\alpha \geq 0$ (cf. (3)).[10]

If, instead, disagreement is not so strong ($K < \theta W^F$), the manager is better off investing immediately as the cost of delay outweighs the potential gains from avoiding dilution by hoarding cash.

Proposition 1 *If disagreement is sufficiently strong ($K > \theta W^F$), it is optimal for the manager to hoard cash and delay the investment. The optimal cash level is given by (7). This threshold is decreasing in θ and ρ—i.e., there is less cash hoarding when the investment opportunity is better and when there is less disagreement.*

What this proposition points at is that delaying is costlier when the investment opportunity is better, implying that the manager will hoard less cash. Furthermore, the cost of delay weighs more when there is less disagreement. Thus, the manager will hoard less cash when there is more alignment with the financier. An important implication of Proposition 1 is that firms with better investment opportunities choose to expand more quickly. This points at a self-reinforcing mechanism leading to an accelerated divergence over time between firms with good and bad investment opportunities.

[10] We omit this bound from our analysis when it does not lead to confusion.

The model we have presented thus far illustrates the idea of non-precautionary hoarding in the simplest possible way, underlining its robustness. This robustness is important, as the implication of non-precautionary hoarding that firms with better investment opportunities hoard less cash is the opposite of what precautionary theories would predict (Bates et al., 2009; Section 3.4). In what follows, we show that non-precautionary hoarding matters for a variety of decisions in the evolution of growth firms. This helps explain stylized facts in the literature that are puzzling from the perspective of existing theories, such as why private firms hoard less cash than public firms and how product market competition affects this choice.

3.2 Cash Hoarding and the Choice Between Public and Private Financing

We now analyze how non-precautionary cash hoarding endogenously depends on the choice between public and private financing. For this purpose, we explore the following trade-off faced by firms at the intersection between public and private financing.[11] On the one hand, the liquidity associated with greater competition among public investors is one of the text-book arguments for public financing. On the other hand, the benefit of private financing is that the financier's closer involvement may help him better understand the business (Ferreira et al., 2014). Furthermore, coordination and negotiations are easier than with dispersed financiers (Brunner and Krahnen, 2008), and managers might be willing to share sensitive information that they might be unwilling to disclose to the public market (Bhattacharya and Ritter, 1983; Maksimovic and Pichler, 2001). All of this could help align priorities and vision and reduce disagreement.

More formally, we assume that there is a higher degree of alignment between a firm and its private financiers: $\rho^{priv} > \rho^{pub}$. Private financiers, however, face an illiquidity cost $\Lambda = L + l\,(K - w)$ for holding an illiquid claim, where $L, l > 0$.[12] Furthermore, we allow

[11] Observe that we are not comparing a typical young firm with no track record with a large and mature public firm about which there is plenty of information. Instead, we are holding the firm's characteristics fixed and analyze its endogenous choice between public and private financing. This is also the comparison that empirical papers are arguably interested in (Gao et al., 2013).

[12] We interpret Λ as the illiquidity cost of private financing net of whatever fee differential there is between public and private financing. On average, public financing fees, such as underwriter, audit, and legal expenses, are 7-10% of IPO proceeds. While this is often more than the 2-10% fees in private placements, it is trumped by the typical illiquidity discount of 20-30% for private deals (Damodaran, 2012). Though it is hard to clearly interpret this discount, as it already incorporates the owner's dilution due to disagreement and financiers' bargaining power, it still suggests that the "net" illiquidity costs Λ can be substantial. Clearly, there would be no trade-off in the present setting if $\Lambda < 0$. However, Section 4.3 extends our results to alternative scenarios in which illiquidity costs are not part of the public-private

private financiers to have bargaining power and extract rents. We model such bargaining power by giving the financiers the right to make a take-it-or-leave it offer to the manager at any given point in time. If the manager rejects this offer, she could go for public financing or return for a new offer at a later point in time. We denote the financier's profit when providing financing for $K-w$ by $\varepsilon(w)$, so that the stake demanded by the financier satisfies

$$\alpha = \frac{(1+l)(K-w) + L + \varepsilon(w)}{\theta W^F(\rho^{priv})}. \tag{8}$$

Somewhat loosely speaking, we can also interpret $\varepsilon(w)$ as the financier's pricing terms. In particular, expression (8) makes it clear that the stake demanded by the private financier depends on his bargaining power. Whether this will lead the firm to hoard more or less cash when choosing private financing compared to public financing depends on the illiquidity costs L and l and how $\varepsilon(w)$ is endogenously set by the financier.

We start by analyzing the role of the financier's bargaining power in shaping the decision to raise public or private financing. The attractiveness of the investment opportunity θ plays a key role. Suppose that θ can take values between $\left[\underline{\theta}, \overline{\theta}\right]$ and let $\Theta \subseteq \left[\underline{\theta}, \overline{\theta}\right]$ be the subset of values for which the manager would prefer private financing if the private financier had no bargaining power (i.e., $\varepsilon(w) = 0$ for all w).

Proposition 2 *(i) Regardless of whether or not the financier has bargaining power, the manager prefers private financing for the same set of values Θ. (ii) The financier's bargaining power does not affect the firm's cash hoarding policy.*

Proposition 2 has important implications. It shows that a private financier's bargaining power will neither change the manager's preference for public or private financing nor her hoarding decision. The first part of Proposition 2 is based on a simple observation. The private financier's bargaining power is restricted by the manager's outside option of choosing public over private financing. Thus, the best the financier can do is extract as much profit as possible from the manager without making her prefer to go for public financing. Intuitively, losing the firm to the public market and losing all rents could never be optimal for the private financier if by slightly reducing the markup, the financier could preserve some positive profit. Hence, the set of values θ for which the firm prefers private financing is unchanged.

Though a private financier can extract rents from the manager for the values in Θ for which the manager strictly prefers private financing, it is important to realize that this is

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trade-off.

done by anticipating the effect of setting higher financing costs on the manager's hoarding and investment decisions. For a manager who is just indifferent between public and private financing, there will be no effect, as the mark-up for such a firm is zero. Even for firms that strictly prefer private financing, the effect is only on profits, and not on the hoarding policy. This is because the best way for the financier to maximize his payoff is to choose the pricing terms $\varepsilon(w)$ in such a way that the manager's and financier's joint surplus is maximal. Distorting the manager's hoarding decision would reduce the surplus that can be shared and, thus, not only impose a cost on the manager, but also on the financier. Hence, the financier maximizes his profits by offering terms for which the manager implements the same hoarding and investment decision as when the financier does not have bargaining power.

Using the insights of Proposition 2, we can show that a firm that chooses to raise private financing hoards less cash than it would hoard when choosing public financing, and that a firm with better investment opportunities is more likely to raise private financing:

Proposition 3 *(i) If a firm prefers private over public financing, it also chooses to hoard less cash $w^{priv} < w^{pub}$. (ii) A firm with better investment opportunities is willing to tolerate higher illiquidity costs and is, thus, more likely to raise private financing.*

To understand the proposition, observe that the time-value-cost of delay is the same regardless of the firm's choice between public and private financing. Hence, this choice is driven by the differential costs and benefits of outside financing. Part (i) follows from the insight that the higher alignment with private financiers, $\rho^{priv} > \rho^{pub}$, is most valuable when there is a substantial need for external financing, in which case also the fixed cost of illiquidity matters less. Hence, private financing is preferred by firms that seek more external financing—i.e., by those that want to invest more quickly and hoard less. By Proposition 2, this insight remains true despite the fact that private financiers extract rents due to their stronger bargaining power. Part (ii) relates to Proposition 1 by pointing out that the firms wanting to invest more quickly are those with better investment opportunities. Thus, they raise private financing and hoard less. Overall, these results could help explain the empirical findings of Gao et al. (2013) and Asker et al. (2015) that private firms hoard less than comparable public firms, which contradicts the predictions of precautionary theories.

Before concluding this section, we would like to stress that our results arise also when considering alternative trade-offs involved in the decision between public and private financing. In Section 4.3, we show the same qualitative predictions when instead of disagreement and illiquidity costs, we consider differential monitoring abilities between public and private financiers.

3.3 Product Market Competition and the Choice Between Public and Private Financing

While competition among financiers has a limited effect on the firm's cash hoarding policy, the same is not true about product market competition. In particular, a common argument against delaying investment is that competition in the product market could lead the firm to lose its first-mover advantage (Grenadier, 2002; Carlson et al., 2006). However, in this section we show that this effect could be dominated, when considering cash hoarding. The reason is that by reducing future profits, competition makes delay and, thus, hoarding less costly. Hence, the net effect of competition on delay and cash hoarding could be positive or negative. This also has an impact on the firm's decision to raise public or private financing.

To show this, we extend the baseline model in the following way: Suppose that the likelihood that a competitor with a similar idea enters the market before the firm invests follows an exponential distribution with parameter λ. This modeling choice is standard in the literature on competition among growth firms (Loury, 1979; Weeds, 2002). The entry parameter $\lambda \in [0, \infty)$ could be interpreted as a measure of the intensity of competition.

Competition reduces profits, including those of the first-mover. This could be because even a first-mover's market share declines with a larger number of competitors or because competition makes differentiation more difficult. We model this by assuming that the first-mover's equilibrium expected payoff after investing is $\pi(\lambda)\theta W^i$, $i \in \{F, M\}$, where $\pi(\lambda) \leq 1$ is differentiable with $\pi'(\lambda) \leq 0$. The equilibrium expected profits if the firm loses its first-mover advantage is given by $\xi(\lambda)\pi(\lambda)\theta W^i$, where $\xi(\lambda) \leq 1$ is differentiable and captures the cost of losing this advantage. Thus, $\pi'(\lambda)$ captures the reduction of profitability due to competition, regardless of whether or not the firm is a first-mover.[13] If this effect dominates the first-mover effect, then competition will increase hoarding. Specifically, the erosion of profits reduces the attractiveness of the investment opportunity and, similar to Proposition 1, makes delaying investment to hoard less costly. By contrast, if the threat of losing the first-mover advantage dominates, competition will accelerate investment—i.e., leading to less hoarding. This is the traditional result that competition

[13] As in the related literature, this reduced-form way of modeling competition captures the main forces behind it. $\xi(\lambda)$ and $\pi(\lambda)$ depend on the way one chooses to model competition after entry. Rather than committing to a specific modeling choice, we limit ourselves to discussing how $\xi(\lambda)$ and $\pi(\lambda)$ affect hoarding. See Grenadier (2002) and Novy-Marx (2007) for models in which firms compete on quantity and try to time market demand.

speeds up investment (Grenadier, 2002).

Proposition 4 *The effect of product market competition on hoarding depends on the relative importance of reduced profitability (regardless of whether or not the firm is a first-mover) and holding on to the first-mover advantage. If the former effect dominates, the firm will delay investment and increase hoarding. By contrast, if retaining a first-mover advantage is more important, the firm will accelerate investment and reduce hoarding.*

In a model of precautionary hoarding, Lyandres and Palazzo (2015) show that the negative effect of competition on profits drives firms to reduce hoarding. This is the opposite result to Proposition 4 and highlights the importance of differentiating between precautionary and non-precautionary hoarding.[14] We can now combine the insights of Propositions 3 and 4 to derive implications for how product market competition and its effect on cash hoarding affect the firm's decision to raise public or private financing.

Proposition 5 *The effect of product market competition on the choice between public and private financing is as follows: (a) If the depressing effect of competition on profitability dominates the negative impact of not being a first-mover, the firm is more likely to raise public financing and expand hoarding. (b) The firm is more likely to raise private financing and accelerate investment if realizing a first-mover advantage is of paramount importance.*

To conclude this section, it is worth mentioning that competition could also drive firms to reduce the size K of investment (which we have assumed to be fixed). A smaller K will mechanically reduce the need for cash. However, this would not change that (for any K) firms with better investment opportunities make investments with a higher proportion of external financing.

3.4 Discussion: Contrasting Non-Precautionary with Precautionary Hoarding

Non-precautionary hoarding describes how much a firm delays investing in an existing investment opportunity to reduce its dependence on dilutive external financing. In this section, we relate this motive to the standard precautionary argument that a firm hoards cash in anticipation of a future growth opportunity (i.e., pre-arrival of an opportunity).

[14] The main novelty in this section is to highlight the dual effect of competition on hoarding and the choice between public and private financing. These results, together with our focus on investment delay, differentiate our paper from Morellec et al. (2015) who show that, by decreasing profits, competition increases the need to hoard cash as an insurance against potential negative illiquidity shocks.

Consider a firm that does not yet have an investment opportunity in $t = 0$, but expects that such an opportunity may present itself at some future point in time. We assume that the time until this event follows an exponential distribution with parameter λ_a. To avoid costly delay following the arrival of the investment opportunity, the manager could start hoarding cash prior to its arrival. This would be optimal if the profitability of the investment opportunity and its probability of arrival are sufficiently large. In this case, the manager sets aside all cash and continues hoarding until the investment opportunity arrives or until she has accumulated sufficient capital to be able to fully finance it without external financing.[15]

Proposition 6 *If the probability of arrival and the profitability of the investment opportunity are sufficiently high, the manager sets aside all her initial cash and continues hoarding until the investment opportunity arrives (or until she has sufficient funds at hand). Upon arrival of the investment opportunity, the manager follows the hoarding and investment policies set out in Propositions 1-5.*

Proposition 6 has several implications. One is that the forces that drive firms to reduce precautionary hoarding, such as a lower profitability of investment opportunities, drive the firm to increase *non-precautionary* hoarding after such opportunity arises. This could also shed light on why precautionary theories fail to explain existing evidence, such as why private firms hoard less cash than comparable public firms.[16]

Another implication of Proposition 6 is that a firm that expects a future growth opportunity will target a specific precautionary cash level. Non-precautionary hoarding can then be understood as describing deviations from this level when the hoarded amount turns out to be insufficient.

4 Extensions and Robustness

In this section we discuss several extensions. First, we argue that non-precautionary hoarding could reveal private information, which could help explain announcement effects when firms decide to raise public financing (Section 4.1). Section 4.2 discusses how cash hoarding and investment decisions are affected when the manager can use debt instead of

[15] Observe that every additional dollar saved is more valuable than the previous one. This is because an additional dollar not only allows to make the investment earlier, but it also reduces the time the already hoarded cash will remain locked-up in the firm prior to undertaking the investment.

[16] Note that we do not have in mind here the (obvious) conclusion that following more precautionary hoarding there is less need for non-precautionary hoarding, but that the incentives are opposite.

equity financing and may pay out cash. Finally, Section 4.3 extends our results to a setting in which the financing friction is an incentive problem ala Holmstrom and Tirole (1997) rather than disagreement. We have also considered the cases in which delaying investment reduces uncertainty and the expected project profitability varies over time, but the insights are unchanged. Details are included in Appendix B.

4.1 Can Cash Hoarding Reveal Growth Prospects?

In this section, we show that our insights on non-precautionary hoarding remain valid in the presence of private information and that hoarding could convey valuable information to financiers regarding a firm's growth prospects. We introduce asymmetric information by making the parameter θ privately known to the manager, but not to financiers. It is common knowledge that θ is drawn from a CDF F on $[\underline{\theta}, \overline{\theta}]$. Let $\widehat{\theta}$ be the financiers' belief about the now unobservable type θ. In (3) we now have $\alpha = \frac{K-w}{\widehat{\theta} W^F}$ and (4)-(6) need to take into account $\widehat{\theta}$—i.e., we write $V\left(w_t, \widehat{\theta}, \theta\right)$ and $U\left(w_t, w^*, \widehat{\theta}, \theta\right)$. Our baseline assumption that the manager has access to a competitive market for capital and can make a take-it-or-leave-it offer to the financier gives rise to a game of signaling, as the manager is privately informed about the firm's type.

An equilibrium candidate in pure strategies for the signaling game can be characterized with a triple of functions $(w_\theta^{**}, \mu^*, \alpha)$, where w_θ^{**} is the cash level that a manager of type θ chooses as target for hoarding; μ^* is the financier's posterior belief that maps w_θ^{**} into the set of probability distributions over the type set $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$; $\alpha \in [0, 1]$ is the minimum equity stake the financier demands in return for funding w_θ^{**}, where we also allow the financier to reject funding. Our equilibrium concept is that of a Perfect Bayesian Equilibrium. To deal with a potential multiplicity of equilibria, the equilibrium set is refined with D1. This standard refinement requires that, upon observing a deviation, the financier restricts his out-of-equilibrium beliefs to the set of types who are most likely to have deviated.

Summarizing, the manager maximizes (5) subject to the condition that the proposed contract is individually rational for a financier who makes zero profit and who uses Bayes rule on the equilibrium path to form his posterior beliefs μ^* when drawing an inference $\widehat{\theta}$ about the firm's type. Note that since the expected cash flow of the investment is linear in θ, we can use $\widehat{\theta} = \int_{\underline{\theta}}^{\overline{\theta}} \theta d\mu^* (\theta)$ to summarize the financier's beliefs about θ.

In a separating equilibrium of the resulting game, the proposed contract must be incentive compatible. More formally, suppose that there is a monotonic differentiable function w^{**}, which outside financiers use to infer the manager's type given her choice of investment threshold. Then, if the manager decides to exercise at $\widehat{w} \in w^{**}\left(\left[\underline{\theta}, \overline{\theta}\right]\right)$, outside financiers infer that the type is $w^{**-1}(\widehat{w})$ and the manager's expected payoff is

$$U(w_t, \widehat{w}, w^{**-1}(\widehat{w}), \theta) = \left(\left(1 - \frac{K - \widehat{w}}{w^{**-1}(\widehat{w}) W^F} \right) \theta W^M - \widehat{w} \right) \left(\frac{w_t}{\widehat{w}} \right)^\beta,$$

which generalizes (6). Since the investment decision must be on the optimal path, w_θ^{**} solves:

$$w_\theta^{**} = \arg \max_{\widehat{w} \in w^{**}([\underline{\theta}, \overline{\theta}])} U(w_t, \widehat{w}, w^{**-1}(\widehat{w}), \theta) \tag{9}$$

where, assuming that a separating equilibrium exists, we evaluate the respective FOC at $w^{**-1}(\widehat{w}) = \theta$. This problem is well-behaved. Lemma A.1 in the Appendix shows that single crossing with respect to cash hoarding holds. Intuitively, while hoarding helps to reduce the dependence on external financing, it is costly (as $\mu < r$) and firms with better investment opportunities face higher costs of delay than firms with worse investment opportunities. At any level of hoarding and for all beliefs $\widehat{\theta}$, the better firms would gain more (or lose less) from reducing hoarding. Hence, delaying is most costly for good types.[17] Given this insight, we can now show that there is a separating equilibrium satisfying the standard equilibrium refinement D1.

Proposition 7 *(i) In a setting with asymmetric information, the manager can separate with the amount of cash she uses to co-finance her investment. In this equilibrium, better types hoard less cash than lower quality types. Furthermore, firms reduce hoarding and delay investment less compared to the symmetric information case. (ii) There is no pooling equilibrium that survives D1.*

To see the intuition behind Proposition 7, recall that disagreement makes it optimal for growth firms to delay investment and hoard cash, but firms with better investment opportunities hoard and delay less. This is the key feature that makes separation possible, as firms with better investment opportunities will signal their types by hoarding less than firms with worse investment opportunities.

Part (ii) of Proposition 7 follows from the fact that a downward deviation from any pooling level of cash hoarding is always preferred by the highest quality type using arguments similar to those underpinning the single crossing result (cf. Lemma A.1 in the Appendix). Thus, by D1 this type would be able to credibly deviate, making it impossible to sustain a pooling equilibrium.

[17] The single crossing property does *not* depend on our assumption that the manager's assessment of the firm's value is linear in θ. It is sufficient that the firm's value is increasing in θ.

The separation result in Proposition 7 implies that our preceding results extend straightforwardly to a setting involving asymmetric information. Furthermore, it implies that the external-to-internal financing mix can help explain announcement effects when growth firms make lumpy investments financed with public financing.

Corollary 1 *The value of a growth firm's stock as assessed by financiers increases in the proportion of external-to-internal financing used for undertaking the investment.*

4.2 Type of Financing and Payouts

Introducing optimal security design in our analysis requires finding the financing contract that would minimize the friction coming from disagreement. That is, from the manager's perspective, the financier should be least sensitive to whether the project turns out to be good (as the manager believes) or bad (as the financier suspects could be the case). This is reminiscent of the intuition in the earlier security design literature (Nachman and Noe, 1994), in which the manager prefers to issue debt since, as a less information-sensitive security, it minimizes underpricing. Intuitively, being able to issue a security that makes the financier's payoff less sensitive to disagreement is similar to having less disagreement in the first place. This analysis, which involves modeling the cash flow process after investment, is relegated to the Appendix.

On payout policies we can be brief. For firms in need of funds to finance growth, payouts worsen the financing problem, making them suboptimal.

Proposition 8 *(i) The manager optimally hoards less cash to co-finance an investment if she has access to (less information-sensitive) debt financing. (ii) A growth firm in need of financing for new investments will not pay out cash in the presence of disagreement.*

We conclude this section by noting that an alternative interpretation of w_t is that it is part of the firm's assets in place that are not plagued by disagreement, moral hazard, or other information frictions. Apart from cash, this could include property, plant and equipment, inventories, and accounts receivables from stable customers, which the firm accumulates over time. Combining the insights from Proposition 8 and Section 3, our results could be more generally interpreted in the context of a growth firm building up information insensitive assets to reduce its dependence on risky external financing (which is marred by various frictions).

4.3 Dilution of Effort Incentives, Non-Precautionary Hoarding, and the Choice Between Public and Private Financing

We have derived our non-precautionary hoarding results assuming that the manager and the financier have different visions. However, non-precautionary hoarding could also be caused by other financing frictions. We have already discussed information asymmetry. Another friction for growth firms is that external financing dilutes ownership and, hence, could reduce the manager's incentives to exert effort, increasing the cost of external financing (Holmstrom and Tirole, 1997).[18]

Specifically, suppose that conditional on being undertaken, the investment succeeds with probability e, in which case it yields θW. With probability $1-e$, it fails and yields zero. Let the success probability e reflect the effort exerted by the manager at cost $\frac{e^2}{2\nu}$ after the investment is undertaken. In addition, assume that financiers monitor the firm, the cost of which we normalize to zero. The difference between public and private financing is then as follows: If the manager chooses private financing, the private financier's advice and monitoring increase the monetary value of the firm by a factor $m > 1$, but decrease the manager's non-monetary private benefits by B (i.e., B is the manager's disutility from being monitored). The same monitoring by (dispersed) public financiers is less effective. It increases firm value by less , normalized to $m^{pub} = 1$, but also reduces the manager's private benefits by less, normalized to $B^{pub} = 0$. These assumptions capture the standard view in the literature, including that on venture capital, that private financiers are more closely involved with monitoring and advising the firm (Schmidt, 2003).

Assuming again equity financing, a private financier demands a stake $\alpha = \frac{K-w^{priv}}{e^{priv}m\theta W}$ to break even. Conditional on investing, the manager's problem is to choose the the optimal level of effort $\widehat{e}$ that maximizes her expected payoff

$$\left(1 - \frac{K - w^{priv}}{e^{priv}m\theta W}\right) m\widehat{e}\theta W - w^{priv} - B - \frac{\widehat{e}^2}{2\nu}. \tag{10}$$

Hence, her effort choice is

$$\widehat{e} = \nu \left(1 - \frac{K - w^{priv}}{e^{priv}m\theta W}\right) m\theta W. \tag{11}$$

Since in equilibrium, we must have $e^{priv} = \widehat{e}$, we obtain

$$e^{priv} = \frac{1}{2}m\nu\theta W + \frac{1}{2}\sqrt{(m\nu\theta W)^2 - 4\nu\left(K - w^{priv}\right)}. \tag{12}$$

[18] We thank Andrey Malenko for suggesting this discussion.

To avoid corner solutions, assume that ν is sufficiently small so that $e^{priv} \leq 1$. We see immediately that the manager's effort e^{priv} is increasing in her co-investment w^{priv}. Furthermore, plugging (11) into (10), we obtain that before investing the manager chooses her optimal hoarding level w^{priv} to solve

$$U^{priv} = \max_{w^{priv}} \left(\frac{(e^{priv})^2}{2\nu} - B - w^{priv} \right) \left(\frac{w_t}{w^{priv}} \right)^{\beta}. \tag{13}$$

Analogous expressions characterize the case with public financing.

It is now straightforward that this alternative setting leads to the same qualitative results as our baseline model. First, it continues to be true that firms with better investment opportunities hoard less, as delay is more costly for them. Second, we have the same conclusions regarding the choice between public and private financing. Intuitively, firms that choose private financing are those for which the boost of the firm's monetary value matters more than the loss of private benefits. Since these are again the firms with better investment opportunities (higher θ), they are more willing to raise private financing. However, they are also willing to invest more quickly and hoard less cash, explaining why (by self-selection), firms choosing private financing hoard less cash.

Proposition 9 *Consider a reformulation of our model as a moral hazard problem in which the advice and monitoring by a private financier increase the investment opportunity's monetary value, but decrease the manager's private benefits. In this setting, non-precautionary hoarding reduces the dilution of the manager's effort incentives, but it continues to be true that firms with better investment opportunities hoard less. Furthermore, firms that choose private over public financing hoard less.*

5 Empirical Implications

We conclude with a discussion of the main empirical implications stemming from our model. Our innovation is to introduce the notion of non-precautionary hoarding and ask: If growth firms have investment opportunities present, but not the funds to finance them, will they delay investment and hoard cash and how does this affect other decisions involved in the evolution of growth firms? Surprisingly, the literature has overlooked that this focus leads to contrasting implications to those stemming from theories on precautionary hoarding. In what follows, we discuss some of these implications. We believe that precautionary hoarding better describes mature firms. Instead, younger growth firms fit better our non-precautionary setting, as these firms often lack the funds to undertake their existing growth opportunities (Hadlock and Pierce, 2010).

Our starting point is to show that firms with better investment opportunities will hoard less. The intuition is as simple as it is robust: Once investment opportunities have arrived, delaying investment to avoid dilution is costlier if the opportunities are better (Proposition 1). By contrast, precautionary hoarding postulates that firms will hoard more if they do so in anticipation of the arrival of profitable growth opportunities (Proposition 6; Opler et al., 1999).

Implication 1: *Having growth prospects triggers cash hoarding. However: (i) Growth firms with better existing investment opportunities hoard less cash, as they want to minimize investment delay (non-precautionary hoarding). (ii) By contrast, firms hoarding in anticipation of good growth opportunities, hoard more (precautionary hoarding). This should be a better description of more mature firms.*

Our analysis implies that cash hoarding is very much dependent on the firm's life-cycle phase. Growth firms with better investment opportunities follow a low cash strategy in their growth phase, but may end up cash rich as they mature. This life-cycle pattern finds support in Drobetz et al. (2015).

Another insight from Proposition 1 is that growth firms with better investment opportunities will finance their investments with a higher proportion of external financing. Combined with the insight that non-precautionary hoarding could signal growth prospects when there are problems of asymmetric information (Proposition 7), this implies:

Implication 2: *Growth firms with better investment opportunities will finance new investment opportunities with a higher fraction of external funds. The stock price reaction to public offerings used to finance growth should be more positive when firms fund investment with a higher proportion of external financing.*

One of the innovations of our paper is to study the interaction of non-precautionary hoarding with the choice between public and private financing. Contrary to precautionary-based theories, which would predict that private firms should hoard more cash as they are more financially constrained, we show that firms choosing private financing engage in less non-precautionary hoarding and delay investments less (Proposition 3).

Implication 3: *A growth firm in a position to choose between public and private financing (i.e., for which the financing choice is an endogenous decision) delays investment less and hoards less cash when choosing private financing.*[19]

Implication 3 finds strong support in a recent empirical study by Gao et al. (2013)

[19] In addition, firms choosing private over public financing are less likely to wait for uncertainty to unravel before investing (see Appendix B).

that explicitly takes into account the endogeneity of the choice between public and private financing. It shows that public firms hoard up to twice as much cash as comparable private firms. Further in line with our theory, Asker et al. (2015) find that private firms not only have less cash, but also react more quickly to new growth opportunities.

In a similar vein, our paper could shed some light on the apparent wish of some firms to gain the best of both world—in particular, the stylized fact that public firms choose private financing when their investment opportunities are better (Gomes and Phillips, 2012; Phillips and Sertsios, 2015). This finding corresponds to the second part of Proposition 3. For such cases, Implication 3 would additionally predict that firms fund investments with a higher proportion of external financing when choosing private over public financing.

Another novel insight of our model concerns the interaction of competition, hoarding, and the choice between public and private financing. Consider, first, the effect on hoarding. The prior literature has focused on the idea that competition accelerates investment, as firms seek to gain a first-mover advantage (Grenadier, 2002). We show that this effect would call for less hoarding. However, we also show that when cash hoarding and investment delay are jointly determined, competition could have the opposite effects: It could lead to investment delay and more hoarding. This occurs if the decrease in profits caused by competition matters more for investment timing than being a first mover (Proposition 4).

Implication 4: *The effects of stronger product market competition are as follows: (i) If having a first-mover advantage is not of paramount importance, growth firms engage in more non-precautionary hoarding, raise less external financing, and delay investment more. (ii) Growth firms follow the opposite strategy if preserving the first-mover advantage is important.*

Implication 4 suggests that in highly competitive industries, where the erosion of profits is significant and first-mover advantages are not sustainable, more competition will induce growth firms to delay investment and increase hoarding. By contrast, in industries where there are still substantial benefits from being a first-mover (as in oligopoly), more competition will make firms speed up investment. This prediction is consistent with Akdogu and MacKay (2008) who document a U-shaped relationship between competition and investment delay—i.e., investment accelerates as competition emerges in highly concentrated industries, but investment slows down when competition increases in already competitive industries.

Consider, next, the effect of competition on the choice between public and private financing.

Implication 5: *The effect of stronger product market competition on the choice be-*

tween public and private financing is as follows: (i) If having a first-mover advantage is not of paramount importance, growth firms are more likely to choose public financing. (ii) Growth firms are more likely to choose private financing if preserving the first-mover advantage is important.

The intuition for Implication 5 is that private financing goes hand-in-hand with (relatively) speedy investment, and this is important when the first-mover advantage is of paramount importance. This result could shed light on the conflicting evidence that some empirical studies find that product market competition leads to more (Chod and Lyandres, 2011) while other to less public financing (Chemmanur et al., 2010).[20]

6 Conclusion

We develop a theory of non-precautionary cash hoarding that analyzes whether a growth firm will choose to delay investments in order to hoard cash and depend less on outside financing. The non-precautionary perspective is one where investment opportunities are already present, but funding is not. This perspective is of primary importance for growth firms, but surprisingly ignored in the literature, which has largely focused on precautionary hoarding. The distinction is far from trivial, as both types of hoarding have very different implications.

In our model, entrepreneurs try to avoid external financing because they are reluctant to see their stake diluted. Our starting point is to show that firms with better investment opportunities hoard less and finance a higher fraction of new investments with outside financing. The key reason is that they find it more costly to delay a more profitable opportunity. By comparison, firms hoarding cash in anticipation of future growth prospects, hoard more when these prospects are better. Thus, the cross-sectional predictions are the opposite. Expanding on this simple insight, we show a number of novel results that question the extent to which standard arguments developed for mature firms (focusing on *precautionary* hoarding) apply to growth firms that seek to satisfy immediate funding needs for investment opportunities at hand (non-precautionary hoarding).

One of our main insights is that growth firms that choose private financing hoard less cash and invest more quickly than they would if they had chosen public financing. This is because firms choose private financing when they value the benefit of a higher alignment

[20] We do not formulate predictions for cash-to-assets ratios. Firms with more investment opportunities would mechanically hoard more cash in our model. Thus, the evidence that firms with a high Tobin's Q hoard more cash (Opler et al., 1999; Bates et al., 2009) could be consistent with both precautionary and non-precautionary motives (see Implication 1).

with financiers more than the costs associated with offering less liquid securities. That is, firms choose private financing when their investment opportunities are better, in which case they delay investment less and, hence, finance new investments with more external financing and less internally generated cash. We show that this is true even if a lack of competition among private financiers makes private financing more expensive. In such cases, financiers will extract rents, but in a way that does not affect investment and hoarding, and also leaves the choice between public and private financing unaffected. This holds because private financiers do not gain from pushing firms to more public financing or more hoarding and, hence, will adjust pricing accordingly. These predictions find support in recent empirical work.

Another prediction is that product market competition can have two opposing effects on hoarding and the choice between public and private financing. The first effect is that competition gives firms an incentive to speed up investment not to lose their first-mover advantage. This will leave less time for hoarding. However, there is a countervailing effect, which could easily dominate: competition is likely to reduce profitability regardless of whether or not the firm is a first-mover. Investment is then less lucrative making delay less costly and hoarding more attractive. Combining these insights with our results on public versus private financing, we predict that, when product market competition strongly erodes profits and drives firms to hoard more cash, firms are more likely to raise public financing. Alternatively, if product market competition leads firms to speed up investment, it will reinforce the benefit of raising private financing.

Several extensions of our model yield further insights into how non-precautionary hoarding affects the evolution of growth firms. For example, we show that asymmetric information actually leads to less hoarding. This is because the hoarding policy conveys a signal about the firm's prospects, which induces firms to choose less hoarding in order to signal a better investment opportunity. Thus, we expect differential announcement effects depending on the internal-to-external financing mix of firms investing in growth opportunities.

Our results also provide new insights on the dynamics of firm evolution and cash holdings. Our analysis focuses on growth firms that are short on cash and operate in an uncertain environment. The ones with the better investment opportunities will choose to grow rapidly using outside funding, and, relative to their lesser peers, will be cash-poor. However, on average, they will be more profitable and successful. This implies that in the follow-up stage after these firms have established themselves, they may start earning cash at a higher rate than needed for investment and growth. High cash holdings are then a sign of past success. This would imply that growth firms striving to become the next Google, Microsoft, or Apple should not try copying the large cash holdings of these already mature firms; as *growth* firms they should follow a very different cash policy.[21] Another implication is that since firms with better opportunities also invest more rapidly, reinforcing effects are present. The result resembles an accelerated Darwinian survival process with "winners taking it all."

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Appendix A Omitted Proofs

Proof of Proposition 1. Differentiating (6) with respect to w^* we obtain

$$\frac{\partial}{\partial w^*}U(w_t, w^*, \theta) = \left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{\theta W^F}\right)\theta W^M - w^*\right) + \left(\frac{\theta W^M}{\theta W^F} - 1\right)\right)\left(\frac{w_t}{w^*}\right)^\beta. \tag{A.1}$$

The first term shows the time-value-loss of waiting for w to increase, while the second term shows the benefit from obtaining cheaper financing when increasing the co-investment. Note that since $\beta > 1$, the right-hand-side (RHS) is negative if $\theta W^F > K$. In this case, hoarding cash is never optimal. Hence, the manager only hoards cash if the disagreement with the financier is sufficiently strong and $\theta W^F < K$. Then, the first order condition (FOC) yields:

$$w^* = \frac{\beta}{\beta - 1}\frac{\left(K - \theta W^F\right)}{\left(\theta W^M - \theta W^F\right)}\theta W^M$$

where note that $\alpha = \frac{K-w}{\theta W^F} \geq 0$ implies that the upper limit of w^* is K and that the manager chooses $w^* = \min\left[\frac{\beta}{\beta-1}\frac{\left(K-\theta W^F\right)}{\left(\theta W^M - \theta W^F\right)}\theta W^M, K\right]$.

For completeness, note that the second order condition is

$$\begin{aligned}\frac{\partial^2}{\partial\left(w^*\right)^2}U(w_t, w^*, \theta) \quad = \quad & \frac{\beta}{\left(w^*\right)^2}\left(\frac{\theta W^F - K}{\theta W^F}\right)\theta W^M\left(\frac{w_t}{w^*}\right)^\beta \\ & -\frac{\beta}{w^*}\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{\theta W^F}\right)\theta W^M - w^*\right) + \left(\frac{\theta W^M}{\theta W^F} - 1\right)\right)\left(\frac{w_t}{w^*}\right)^\beta.\end{aligned}$$

At the internal optimum (when the FOC holds), which is the case when $\theta W^F < K$, the second line on the RHS is zero, and the expression is negative. Clearly, if the initial cash at hand is $w_0 > w^*$, the manager invests immediately. Finally, observe that w^* decreases in θ.[22] **Q.E.D.**

Proof of Proposition 2. We show first that the set of values θ for which the manager prefers private financing is unchanged by the financier's price setting power (Step 1). We then describe how the financier chooses $\varepsilon(w)$, which we refer to as his pricing schedule,

[22] This is true even if we would take a more general functional form for the expected payoff of the firm as a function of θ, $W^{F,M}(\theta)$, as long as it is monotonically increasing in θ.

and how he internalizes potential financing distortions (Step 2).

Step 1: We argue to contradiction. Suppose that the financier sets a pricing schedule $\varepsilon(w) > 0$ for which the manager is better off with public financing. By sticking to this offer, the financier makes zero profit and, thus, has incentives to deviate to offering more favorable financing terms for the manager. In particular, the private investor will have incentives to deviate from any pricing schedule for which the manager's expected payoff with private financing is less attractive than her outside option of public financing as long as by doing so he can achieve $\varepsilon(w) \geq 0$. Thus, if it is profitable for a firm to raise private financing when the private financier has no bargaining power (i.e., $\varepsilon = 0$), the private financier's offer will be such that she raises private financing also if he has such power.

Step 2: We conclude by formalizing the claim that the financier's bargaining power does not lead to hoarding distortions.[23]

At any point in time he might be approached by the manager, the private financier faces the following problem: He chooses a pricing schedule $\varepsilon(w) \geq 0$, which sets the markup for any given cash level hoarded by the firm (respectively for any given amount $K - w$ that the firm is trying to raise). This pricing schedule maximizes his expected payoff $\varepsilon(w^{priv})\left(\frac{w}{w^{priv}}\right)^{\beta}$ subject to the constraints the the manager optimally chooses how much cash to hoard w^{priv} and that the manager is better off raising private financing than with her outside option of raising public financing. In analogy to (6), the manager's expected payoff is

$$\left(\left(1 - \frac{(1+l)(K - w^{priv}) + L + \varepsilon(w^{priv})}{\theta W^F}\right)\theta W^M - w^{priv}\right)\left(\frac{w}{w^{priv}}\right)^{\beta} \tag{A.2}$$

implying that w^{priv} is pinned down by the first-order-condition

$$\begin{aligned} 0 \quad = \quad & -\frac{\beta}{w^{priv}}\left(\left(1 - \frac{(1+l)(K - w^{priv}) + L + \varepsilon(w^{priv})}{\theta W^F}\right)\theta W^M - w^{priv}\right) \\ & + \left(-\frac{\frac{\partial \varepsilon(w^{priv})}{\partial w^{priv}} - (1+l)}{\theta W^F}\theta W^M - 1\right). \end{aligned} \tag{A.3}$$

This condition gives the optimal cash hoarding policy for a given pricing schedule $\varepsilon(\cdot)$. We can now express condition (A.3) as a differential equation for $\varepsilon(\cdot)$ of the form

$$\varepsilon(\widetilde{w}) - \frac{\partial \varepsilon(\widetilde{w})}{\partial \widetilde{w}}\frac{\widetilde{w}}{\beta} + \left((1+l)K + L - \theta W^F\right) - \frac{(\beta - 1)}{\beta}\widetilde{w}\left(\frac{(1+l)\theta W^M - \theta W^F}{\theta W^M}\right) = 0, \tag{A.4}$$

[23] Later we show that asymmetric information could lead firms to reduce hoarding, but this does not change the intuition and the qualitative results in this section.

which defines how the private financier should set the pricing schedule $\varepsilon(\cdot)$ if he is seeking to influence the manager's decision to choose any given hoarding strategy $\widetilde{w}$. The differential equation (A.4) together with the appropriate boundary condition, which we give towards the end of the proof, pin down the shape of $\varepsilon(\cdot)$. Using this, it is convenient to restate the financier's problem as that of choosing

$$w^{priv} = \arg\max_{\widetilde{w}} \varepsilon(\widetilde{w}) \left(\frac{w}{\widetilde{w}}\right)^{\beta}. \tag{A.5}$$

giving the first order condition

$$\left(\frac{\partial \varepsilon(w^{priv})}{\partial w^{priv}} - \varepsilon\left(w^{priv}\right) \frac{\beta}{w^{priv}}\right) \left(\frac{w}{w^{priv}}\right)^{\beta} = 0. \tag{A.6}$$

Combining (A.6) with the manager's first-order-condition (A.3), we obtain

$$0 = -\frac{\beta}{w^{priv}} \left(\left(1 - \frac{(1+l)(K - w^{priv}) + L}{\theta W^F}\right) \theta W^M - w^{priv}\right) + \left(\frac{(1+l)\theta W^M}{\theta W^F} - 1\right)$$

giving the same solution for w^{priv} that would obtain if ε were zero for all w. Finally, to pin down $\varepsilon(\cdot)$, we impose the boundary condition that the financier sets the mark-up in a way that the manager is just indifferent between public and private financing at her optimal hoarding level w^{priv}: $U^{priv}(w^{priv}, w^{priv}, \theta) = U^{pub}\left(w^{priv}, w^{pub}, \theta\right)$ (in Proposition 3, we verify that $w^{priv} \leq w^{pub}$). This allows the financier to extract as much rent as possible from the manager without driving her to raise public financing.

Since the above arguments are true at each point in time the manager could approach the financier, we obtain that the financier's pricing schedule does not distort investment timing (and, thus, cash hoarding) relative to the case when financiers have no bargaining power. In a previous working paper version, we have shown that the same insight characterizes the case in which the financier could lose his price setting power with a positive probability at any given point in time. **Q.E.D.**

Proof of Proposition 3. In analogy to Proposition 1, a manager raising private financing solves the FOC

$$-\frac{\beta}{w^{priv}} \left(\left(1 - \frac{L + (1+l)(K - w^{priv})}{\theta W^F(\rho^{priv})}\right) \theta W^M - w^{priv}\right) + \left(\frac{1+l}{\theta W^F(\rho^{priv})} \theta W^M - 1\right), \tag{A.7}$$

while a manager raising public financing solves

$$-\frac{\beta}{w^{pub}}\left(\left(1-\frac{K-w^{pub}}{\theta W^{F}\left(\rho^{pub}\right)}\right)\theta W^{M}-w^{priv}\right)+\left(\frac{\theta W^{M}}{\theta W^{F}\left(\rho^{pub}\right)}-1\right). \tag{A.8}$$

Consider a firm that is just indifferent between public and private financing. For this firm, it holds

$$\begin{aligned}
0 &= U^{priv}(w_t, w^{priv}, \theta) - U^{pub}(w_t, w^{pub}, \theta) \\
&= \left(\left(1-\frac{L+(1+l)\left(K-w^{priv}\right)}{\theta W^{F}\left(\rho^{priv}\right)}\right)\theta W^{M}-w^{priv}\right)\left(\frac{w_t}{w^{priv}}\right)^{\beta} \\
&\quad -\left(\left(1-\frac{K-w^{pub}}{\theta W^{F}\left(\rho^{pub}\right)}\right)\theta W^{M}-w^{pub}\right)\left(\frac{w_t}{w^{pub}}\right)^{\beta} \\
&= \left(\frac{(1+l)\,\theta W^{M}}{\theta W^{F}\left(\rho^{priv}\right)}-1\right)\frac{w^{priv}}{\beta}\left(\frac{w_t}{w^{priv}}\right)^{\beta}-\left(\frac{\theta W^{M}}{\theta W^{F}\left(\rho^{pub}\right)}-1\right)\frac{w^{pub}}{\beta}\left(\frac{w_t}{w^{pub}}\right)^{\beta} \qquad \text{(A.9)} \\
&= \left(\left(\frac{(1+l)\,W^{M}}{W^{F}\left(\rho^{priv}\right)}-1\right)\left(\frac{w^{pub}}{w^{priv}}\right)^{\beta-1}-\left(\frac{W^{M}}{W^{F}\left(\rho^{pub}\right)}-1\right)\right)\frac{\left(w_t\right)^{\beta}}{\beta\left(w^{pub}\right)^{\beta-1}} \qquad \text{(A.10)}
\end{aligned}$$

where the third equality (A.9) follows from the manager's first-order-conditions (A.7) and (A.8). Thus, we have that $w^{pub} \geq w^{priv}$ if and only if

$$\left(\frac{(1+l)\,W^{M}}{W^{F}\left(\rho^{priv}\right)}-1\right) \leq \left(\frac{W^{M}}{W^{F}\left(\rho^{pub}\right)}-1\right) \Longleftrightarrow (1+l) \leq \frac{W^{F}\left(\rho^{priv}\right)}{W^{F}\left(\rho^{pub}\right)} \tag{A.11}$$

Note that (A.10) depends on θ, as the optimal choices of w^{priv} and w^{pub} depend on θ.

To see that (A.11) must indeed be satisfied if the firm chooses private financing, suppose instead that we had $(1+l) > \frac{W^{F}\left(\rho^{priv}\right)}{W^{F}\left(\rho^{pub}\right)}$. For any w^{*}, this would imply

$$\begin{aligned}
\alpha = \frac{L+(1+l)\left(K-w^{*}\right)}{\theta W^{F}\left(\rho^{priv}\right)} &> \frac{L}{\theta W^{F}\left(\rho^{priv}\right)}+\frac{\left(K-w^{*}\right)}{\theta W^{F}\left(\rho^{pub}\right)} \\
&\geq \frac{\left(K-w^{*}\right)}{\theta W^{F}\left(\rho^{pub}\right)}. \qquad \text{(A.12)}
\end{aligned}$$

where the last line (A.12) is the stake the firm would have given to a public financier when raising financing for the same amount w^{*}, implying that public financing would be strictly preferred, giving a contradiction. Thus, condition (A.11) must hold in any formulation of our model in which the manager chooses private financing and we have $w^{pub} \geq w^{priv}$. To see that the inequality must be strict, suppose to a contradiction that $w^{pub} = w^{priv}$, implying that (A.11) is satisfied with equality. We then have

$$\begin{aligned}\alpha = \frac{L+(1+l)(K-w^*)}{\theta W^F(\rho^{priv})} &= \frac{L}{\theta W^F(\rho^{priv})} + \frac{(K-w^*)}{\theta W^F(\rho^{pub})} \\ &> \frac{(K-w^*)}{\theta W^F(\rho^{pub})}. \end{aligned} \tag{A.13}$$

contradicting that the manager is indifferent between raising $w^* = w^{priv} = w^{pub}$ from a public or private financier.

(ii) The manager is indifferent between public and private financing if

$$\Delta_U := U^{priv}(w_t, w^{priv}, \theta) - U^{pub}(w_t, w^{pub}, \theta) = 0.$$

Since $\frac{d\Delta_U}{dl} < 0$ (and $\frac{d\Delta_U}{dL} < 0$; note that by the Envelope Theorem we can ignore the effect of l and L on w^{priv}), there is a level for the proportional illiquidity cost parameter l (and, respectively L), for which this equality can be satisfied for any given θ. Let $\bar{l}$ and $\overline{L}$ denote these threshold levels. Since $\frac{d\bar{l}}{d\theta} = -\frac{\partial\Delta_U}{\partial\theta} / \frac{\partial\Delta_U}{\partial l}$ and $\frac{d\overline{L}}{d\theta} = -\frac{\partial\Delta_U}{\partial\theta} / \frac{\partial\Delta_U}{\partial L}$, we only need to sign $\frac{\partial\Delta_U}{\partial\theta}$. It holds

$$\begin{aligned}\frac{\partial\Delta_U}{\partial\theta} &= \frac{\partial}{\partial\theta}\left(\begin{array}{c}\left(\left(\theta W^F(\rho^{priv}) - (L+(1+l)(K-w^{priv}))\right)\frac{W^M}{W^F(\rho^{priv})} - w^{priv}\right)\left(\frac{w_t}{w^{priv}}\right)^\beta \\ -\left(\left(\theta W^F\left(\rho^{pub}\right) - \left(K-w^{pub}\right)\right)\frac{W^M}{W^F\left(\rho^{pub}\right)} - w^{pub}\right)\left(\frac{w_t}{w^{pub}}\right)^\beta\end{array}\right) \\ &= W^M\left(\left(\frac{w^{pub}}{w^{priv}}\right)^\beta - 1\right)\left(\frac{w_t}{w^{pub}}\right)^\beta > 0,\end{aligned}$$

where we use that $w^{pub} > w^{priv}$. Hence, $\frac{\partial\bar{l}}{\partial\theta} > 0$ and $\frac{\partial\overline{L}}{\partial\theta} > 0$, implying that for higher θ, the manager is more likely to raise private financing even for a higher level of proportional illiquidity cost l. In that case, we equivalently have that for any given l and L, there is a cutoff θ_Λ, such that the manager chooses private financing if and only if the firm's profitability is higher than this cutoff.

Proof of Proposition 4. **Step 1**. We show first that a firm that could still enjoy a first-mover advantage hoards less cash than it would hoard as a second-mover. Let w^*_{SM} denote the firm's optimal hoarding level as a second-mover. After a competitor arrives and becomes the first-mover, the manager's expected payoff can be derived analogously to

Proposition 1 as

$$\overline{U}\left(w_t, w^*\right) = \left(\left(1 - \frac{K - w^*_{SM}}{\xi\pi\theta W^F}\right)\xi\pi\theta W^M - w^*_{SM}\right)\left(\frac{w_t}{w^*_{SM}}\right)^{\beta}, \tag{A.14}$$

where for notational simplicity we omit in what follows the dependence of ξ and π on the competition variable λ. By the same arguments as above, we have that $w^*_{SM} = \frac{\beta}{\beta-1}\left(K - \xi\pi\theta W^F\right)\frac{W^M}{(W^M - W^F)}$.

Let w^*_{FM} be the manager's optimal cash hoarding level while still being a potential first-mover. Applying Ito's lemma modified for jump processes we have

$$rU\left(w\right) = \mu w U_w\left(w\right) + \frac{1}{2}\sigma^2 w^2 U_{ww}\left(w\right) + \lambda\left(\overline{U}\left(w\right) - U\left(w\right)\right). \tag{A.15}$$

In what follows, we argue to a contradiction that $w^*_{FM} < w^*_{SM}$. Suppose not. If $w^*_{SM} < w^*_{FM}$, we have two cases. If $w^*_{SM} \leq w \leq w^*_{FM}$, the manager's expected payoff takes the form $A_1 w^{\beta_1} + B_1 w^{\beta_2} + C_1 w + D_1$, where β_1 and β_2 are the positive and, respectively, negative root of $(r+\lambda) - \mu y - \frac{1}{2}\sigma^2 y\left(y-1\right) = 0$, and where observe that $\frac{\partial}{\partial\lambda}\beta_1 > 0$ implies that $\beta_1 > \beta$. Using the value matching condition $U_{FM_1}\left(w^*_{FM}\right) = \left(1 - \frac{K - w^*_{FM}}{\pi\theta W^F}\right)\pi\theta W^M - w^*_{FM}$ to derive A_1 and (A.15) to derive C_1 and D_1, the manager's expected payoff when $w^*_{SM} \leq w \leq w^*_{FM}$ is

$$\begin{aligned} U_{FM_1}\left(w\right) &= A_1 w^{\beta_1} + B_1 w^{\beta_2} + C_1 w + D_1 \\ &= \begin{pmatrix} \left(1 - \frac{K - w^*_{FM}}{\pi\theta W^F}\right)\pi\theta W^M - w^*_{FM} - B_1\left(w^*_{FM}\right)^{\beta_2} \\ -\frac{\lambda}{r+\lambda-\mu}\left(\frac{W^M}{W^F} - 1\right)w^*_{FM} - \frac{\lambda}{r+\lambda}\left(1 - \frac{K}{\pi\theta W^F}\right)\pi\theta W^M \end{pmatrix}\left(\frac{w}{w^*_{FM}}\right)^{\beta_1} \\ &\quad + B_1 w^{\beta_2} + \frac{\lambda}{r+\lambda-\mu}\left(\frac{W^M}{W^F} - 1\right)w + \frac{\lambda}{r+\lambda}\left(1 - \frac{K}{\xi\pi\theta W^F}\right)\xi\pi\theta W^M \end{aligned}$$

If, instead, $w \leq w^*_{SM} \leq w^*_{FM}$, then the manager's expected payoff takes the form $A_2 w^{\beta_1} + B_2 w^{\beta_2} + C_2 w^{\beta}$. Since the option value of investing is zero for $w \to 0$, we must have $B_2 = 0$. Furthermore, using (A.15) to derive C_2, we have

$$U_{FM_2}\left(w\right) = A_2 w^{\beta_1} + \left(\left(1 - \frac{K - w^*_{SM}}{\xi\pi\theta W^F}\right)\xi\pi\theta W^M - w^*_{SM}\right)\left(\frac{w}{w^*_{SM}}\right)^{\beta}.$$

Finally, we can obtain A_2 and B_1 from the value matching condition $U_{FM_1}\left(w^*_{SM}\right) = U_{FM_2}\left(w^*_{SM}\right)$, and the smooth pasting condition $\frac{\partial}{\partial w}U_{FM_1}\left(w\right)|_{w=w^*_{SM}} = \frac{\partial}{\partial w}U_{FM_2}\left(w\right)|_{w=w^*_{SM}}$. In particular, expressing A_2 from the first condition to plug into the second, we obtain B_1

after some reformulations as

$$B_1 = \frac{1}{\beta_1 - \beta_2} \left(-\beta_1 \frac{\beta - 1}{\beta} \frac{r}{r + \lambda} + (\beta_1 - 1) \left(\frac{r - \mu}{r + \lambda - \mu} \right) \right) \left(\frac{W^M}{W^F} - 1 \right) \frac{w_{SM}^*}{(w_{SM}^*)^{\beta_2}}$$

Suppose now that $w_{SM}^* \leq w \leq w_{FM}^*$. After some transformations of the first order condition of $U_{FM_1}(w)$ with respect to w_{FM}^*, we obtain

$$\begin{aligned} w_{FM}^* &= \frac{\beta_1 \frac{r}{r+\lambda} \left(K - \pi\theta W^F \right) \frac{\pi\theta W^M}{\pi\theta W^F} + (\beta_1 - \beta_2) B_1 (w_{FM}^*)^{\beta_2}}{(\beta_1 - 1) \frac{r-\mu}{r+\lambda-\mu} \left(\frac{W^M}{W^F} - 1 \right)} \\ &< \frac{\beta_1 \frac{r}{r+\lambda} \left(K - \xi\pi\theta W^F \right) \frac{\pi\theta W^M}{\pi\theta W^F} + (\beta_1 - \beta_2) B_1 (w_{FM}^*)^{\beta_2}}{(\beta_1 - 1) \frac{r-\mu}{r+\lambda-\mu} \left(\frac{W^M}{W^F} - 1 \right)} \\ &= \frac{\beta_1 \frac{r}{r+\lambda} \frac{\beta-1}{\beta} + \left(-\beta_1 \frac{\beta-1}{\beta} \frac{r}{r+\lambda} + (\beta_1 - 1) \left(\frac{r-\mu}{r+\lambda-\mu} \right) \right) \left(\frac{w_{FM}^*}{w_{SM}^*} \right)^{\beta_2}}{(\beta_1 - 1) \frac{r-\mu}{r+\lambda-\mu}} w_{SM}^*. \quad \text{(A.16)} \end{aligned}$$

Observe that $\left(-\beta_1 \frac{\beta-1}{\beta} \frac{r}{r+\lambda} + (\beta_1 - 1) \left(\frac{r-\mu}{r+\lambda-\mu} \right) \right)$ is positive (as it is zero for $\lambda = 0$ and its first derivative is positive at $\lambda = 0$). Expression (A.16) implies then

$$\left(\frac{w_{FM}^*}{w_{SM}^*} \right)^{\beta_2} > \frac{-\beta_1 \frac{r}{r+\lambda} \frac{\beta-1}{\beta} + (\beta_1 - 1) \frac{r-\mu}{r+\lambda-\mu} \frac{w_{FM}^*}{w_{SM}^*}}{\left(-\beta_1 \frac{\beta-1}{\beta} \frac{r}{r+\lambda} + (\beta_1 - 1) \left(\frac{r-\mu}{r+\lambda-\mu} \right) \right)} > \frac{w_{FM}^*}{w_{SM}^*},$$

which is a contradiction to $w_{FM}^* > w_{SM}^*$ since $\beta_2 < 0$. Thus, it must be that $w_{FM}^* \leq w_{SM}^*$.

Step 2. We now analyze the effect of λ on w_{FM}^*. Since $w \leq w_{FM}^* \leq w_{SM}^*$, the manager's expected payoff takes the form $Aw^{\beta_1} + Bw^{\beta_2} + Cw^{\beta}$. We have again $B = 0$, since the option to invest is zero for $w \to 0$, and it can be verified that

$$U_{FM} = \overline{U}(w, w_{SM}^*) + \left(\left(1 - \frac{K - w_{FM}^*}{\pi\theta W^F} \right) \pi\theta W^M - w_{FM}^* - \overline{U}(w_{FM}^*, w_{SM}^*) \right) \left(\frac{w}{w_{FM}^*} \right)^{\beta_1}$$

where, plugging in from (A.14), w_{FM}^* is the solution to the first-order-condition

$$0 = -\frac{\beta_1}{w_{FM}^*} \left(1 - \frac{K}{\pi\theta W^F} \right) \pi\theta W^M + \left(\frac{W^M}{W^F} - 1 \right) (1 - \beta_1) + \frac{(\beta_1 - \beta)}{w_{FM}^*} \overline{U}(w_{FM}^*, w_{SM}^*).$$

By standard monotone comparative statics arguments, the dependence of w_{FM}^* on λ is given by the cross partial $\frac{\partial^2 U_{FM_1}}{\partial w^*_{FM}\partial\lambda}$, which after some transformations becomes

$$-\frac{1}{w^*_{FM}}\frac{\partial\beta_1}{\partial\lambda}\left(\left(1-\frac{K-w^*_{FM}}{\pi\theta W^F}\right)\pi\theta W^M - w^*_{FM} - \overline{U}\left(w^*_{FM}, w^*_{SM}\right)\right)\left(\frac{w^*_{FM}}{w^*_{SM}}\right)^{\beta_1}$$
$$+\frac{\beta_1}{w^*_{FM}}\left(-\frac{\partial}{\partial\lambda}\pi+\frac{\beta_1-\beta}{\beta_1}\frac{\partial}{\partial\lambda}\left(\xi\pi\right)\left(\frac{w^*_{FM}}{w^*_{SM}}\right)^{\beta}\right)\theta W^M\left(\frac{w_t}{w^*_{FM}}\right)^{\beta_1}$$

The first line of this expression reflects the firm's cost due to higher competition, reducing the likelihood of being a first-mover. This line is negative as $\frac{\partial\beta_1}{\partial\lambda} > 0$—i.e., this cost calls for speeding up investment. The second line reflects the decrease in expected payoffs due to higher competition. If $\xi'\left(\lambda\right) \geq 0$, this line is positive, as $\frac{\partial\pi}{\partial\lambda} < 0$ and because $\frac{\beta_1-\beta}{\beta_1}$ and $\left(\frac{w^*_{FM}}{w^*_{SM}}\right)^{\beta}$ and ξ are all less than one. If $\xi'\left(\lambda\right) < 0$, the positive effect remains true as long as $\xi'\left(\lambda\right)$ is not too negative—i.e., as long as the decrease in profit ξ due to losing the firm's first-mover advantage does not become increasingly strong for higher levels of competition λ. Thus, overall, cash hoarding decreases if the negative effect of losing the firm's first-mover advantage is strong, and increases otherwise. **Q.E.D.**

Proof of Proposition 6. Observe first that the expected value of the investment opportunity upon its arrival is $U(w^*_{0,\theta}, w^*_{\theta})$, where U is given by (6) from Section 2 and $w^*_{0,\theta}$ is the cash level that the manager has hoarded before arrival. It is straightforward to verify that $U(w^*_{0,\theta}, w^*_{\theta})$ is strictly increasing and convex in $w^*_{0,\theta}$.[24] Suppose now that it is optimal to stop hoarding before arrival and before the manager has hoarded K—i.e., $w^*_{0,\theta} < K$. We argue to a contradiction that this cannot be the case.

Suppose that before arrival, having reached $w^*_{0,\theta}$, the hoarded amount w_t increases above $w^*_{0,\theta}$. Paying out $w_t - w^*_{0,\theta}$ cannot be optimal if hoarding until $w^*_{0,\theta}$ is optimal. First, the probability of arrival is the same at every instant. Second (given the convexity of U), the marginal increase in the option value U that the manager would have after arrival is increasing in the hoarded amount before arrival. In contrast, paying out a unit of cash has the same value to the manager regardless of the previously hoarded amount. Hence, if hoarding dominates paying out for $w_t < w^*_{0,\theta}$, it is more beneficial also for $w_t > w^*_{0,\theta}$.

To determine whether the manager should start hoarding, we have to compare the expected payoff from hoarding as prescribed above with paying out w_0. Clearly, this expected payoff must be increasing in θ and the probability of arrival λ_a. Hence, there is a threshold $\overline{\lambda}_a\left(\theta\right)$, above which setting aside w_0 and hoarding is optimal for type θ;

[24] This is intuitive, as upon arrival of the investment opportunity, the manager has the (call) option to invest.

respectively, there is a threshold $\overline{\theta}(\lambda)$, such that hoarding is optimal if $\theta > \overline{\theta}(\lambda)$. In this case, the manager hoards until the arrival of the investment opportunity and, upon arrival, follows Propositions 1. Note that the manager will stop hoarding cash once she becomes independent of external financing.[25] **Q.E.D.**

Before proving Proposition 7, we start by showing a useful result.

Lemma A.1 *Single crossing holds because*

$$\frac{\partial}{\partial\theta}\left(-\frac{\frac{\partial}{\partial w^*}U}{\frac{\partial}{\partial\widehat{\theta}}U}\right) > 0, \tag{A.17}$$

where $\widehat{\theta}$ *is the financier's inference about the firm's type* θ.

Proof of Lemma A.1. Let for this proof $W^M(\theta)$ and $W^F(\theta)$ denote more generically the manager's and the financier's assessment of the firm's value after investment (as a function of θ), where in the main text we have $W^F(\theta) = \theta W^F$ and $W^M(\theta) = \theta W^M$. Single crossing holds as

$$\begin{aligned}
&\frac{\partial}{\partial\theta}\left(-\frac{\frac{\partial}{\partial w^*}U\left(w_t, w^*, \widehat{\theta}, \theta\right)}{\frac{\partial}{\partial\widehat{\theta}}U\left(w_t, w^*, \widehat{\theta}, \theta\right)}\right)\\
= \;&\frac{\partial}{\partial\theta}\left(-\frac{\left(-\frac{\beta}{w^*}\left(\left(1-\frac{K-w^*}{W^F(\widehat{\theta})}\right)W^M(\theta)-w^*\right)+\frac{W^M(\theta)}{W^F(\widehat{\theta})}-1\right)\left(\frac{w_t}{w^*}\right)^{\beta}}{\frac{K-w^*}{\left(W^F(\widehat{\theta})\right)^2}\frac{\partial}{\partial\widehat{\theta}}W^F(\widehat{\theta})W^M(\theta)\left(\frac{w_t}{w^*}\right)^{\beta}}\right)\\
= \;&\frac{1}{\frac{K-w^*}{\left(W^F(\widehat{\theta})\right)^2}\frac{\partial}{\partial\widehat{\theta}}W^F(\widehat{\theta})}\left(\frac{(\beta-1)\frac{\partial}{\partial\theta}W^M(\theta)}{\left(W^M(\theta)\right)^2}\right) > 0.
\end{aligned} \tag{A.18}$$

Thus, as claimed in footnote 17, for single crossing it is sufficient that the manager's and the financiers' assessments of firm value are increasing in θ (and not necessarily linear in θ). **Q.E.D.**

Proof of Proposition 7. We show first the existence of a separating equilibrium. Then we show that there is no pooling equilibrium that survives D1.

[25] To avoid the risk that the cash at hand falls below K, she may hoard slightly more than K before starting to pay out. Furthermore, note that if the manager does not start hoarding, she pays out w_0 and then cannot invest upon arrival.

Claim 1. *There exists a separating equilibrium*

To show existence of a separating equilibrium, we follow standard arguments.[26] Rewriting (9), we obtain

$$w_{\theta}^{**} = \arg \max_{\widehat{w} \in w^{**}\left([\underline{\theta}, \overline{\theta}]\right)} \left(\left(1 - \frac{K - \widehat{w}}{w^{**-1}(\widehat{w}) W^F}\right) \theta W^M - \widehat{w} \right) \left(\frac{w_t}{\widehat{w}}\right)^{\beta}$$

Taking the FOC and assuming that a separating equilibrium exists—i.e., $w^{**-1}(\widehat{w}) = \theta$–we have

$$\frac{dw_{\theta}^{**}}{d\widehat{\theta}} = \frac{-\frac{\partial}{\partial \widehat{\theta}} U(w_t, w_{\widehat{\theta}}^{**}, \widehat{\theta}, \theta)|_{\widehat{\theta}=\theta}}{-\frac{\beta}{w_{\theta}^{**}} \left(\left(1 - \frac{K - w_{\theta}^{**}}{\widehat{\theta} W^F}\right) \theta W^M - w_{\theta}^{**} \right) \left(\frac{w_t}{w_{\theta}^{**}}\right)^{\beta} + \left(\frac{\theta W^M}{\widehat{\theta} W^F} - 1\right) \left(\frac{w_t}{w_{\theta}^{**}}\right)^{\beta}} . \tag{A.19}$$

To solve this equation we need the appropriate boundary condition. Since a high type has no incentive to mimic low types, we can set: $w_{\underline{\theta}}^{**} = w_{\underline{\theta}}^{*}$, where w_{θ}^{*} is obtained from expression (7) for $\theta = \underline{\theta}$. We can now apply Theorems 1-3 from Mailath (1987) to prove the proposition (in Appendix B we verify that the conditions for these theorems are satisfied). From these theorems it follows that there is a unique separating equilibrium in which w_{θ}^{**} is continuous and differentiable, satisfies (A.19), and $\frac{dw_{\theta}^{**}}{d\theta} < 0$ ($\frac{dw_{\theta}^{**}}{d\theta}$ has the same sign as $\frac{\partial^2}{\partial \widehat{w} \partial \theta} U(w_t, \widehat{w}, \widehat{\theta}, \theta)$).

We now show that $w_{\theta}^{**} < w_{\theta}^{*}$. To see this, rewrite (A.19) as

$$\begin{aligned} & -\frac{\beta}{w_{\theta}^{**}} \left(\left(1 - \frac{K - w_{\theta}^{**}}{w^{**-1}(\widehat{w}) W^F}\right) \theta W^M - w_{\theta}^{**} \right) \left(\frac{w_t}{w_{\theta}^{**}}\right)^{\beta} + \left(\frac{\theta W^M}{w^{**-1}(\widehat{w}) W^F} - 1\right) \left(\frac{w_t}{w_{\theta}^{**}}\right)^{\beta} \\ = & \frac{-\frac{\partial}{\partial \widehat{\theta}} U(w_t, w_{\widehat{\theta}}^{**}, \widehat{\theta}, \theta)}{\frac{dw_{\theta}^{**}}{d\widehat{\theta}}} . \end{aligned} \tag{A.20}$$

Compare (A.20) to the optimality condition (A.1) in Proposition 1. The RHS of (A.20) is positive, while it is zero absent information asymmetry. Thus, taking into account that the LHS decreases in w_{θ}^{**}, we must have $w_{\theta}^{**} < w_{\theta}^{*}$.

Claim 2. *There is no pooling equilibrium that survives D1*

Suppose that there is a pooling equilibrium in which all types pool at a cash level $w^P > w_0$. $\widehat{\theta}$ is then simply $\widehat{\theta} = \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)$. We start by defining D1 in the context of this game.

[26] For a detailed general analysis on this point, see Mailath (1987) and for separation in the context of real options see Grenadier and Malenko (2011) and Morellec and Schürhoff (2011).

Definition 1 *For every deviation $\widetilde{w}$, determine for every type the highest equity stake $\widetilde{\alpha}(\theta)$, for which the deviation payoff $\widetilde{U}(w_t, \widetilde{w}, \widetilde{\alpha}, \theta)$ is higher than the equilibrium expected payoff $U\left(w_t, w^P, \alpha^P, \theta\right)$*

$$\widetilde{\alpha}(\theta) = \arg\min_{\widehat{\alpha}} \left\{ \widetilde{U}(w_t, \widetilde{w}, \widehat{\alpha}, \theta) \,|\, \widetilde{U}(w_t, \widetilde{w}, \widehat{\alpha}, \theta) \geq U\left(w_t, w^P, \alpha^P, \theta\right) \right\}$$

Then, D1 requires that the financier believes that the deviation comes from the types who find $\widetilde{w}$ attractive for the highest equity stake $A \in \arg\max_\theta \widetilde{\alpha}(\theta)$.

An equilibrium candidate does not survive D1 if there is a deviation by some type θ' for which type θ''s minimum expected payoff following deviation is higher then her equilibrium expected payoff, where investors form their out-of-equilibrium beliefs as described in Definition 1 and play a best response. A best response in a competitive market is to demand a stake for which the investor breaks even given his out-of-equilibrium beliefs.

Suppose that the investor observes a downward deviation $\widetilde{w} < w^P$. We have

$$\widetilde{\alpha}(\theta) = 1 - \frac{\left(\left(1-\alpha^P\right)\theta W^M - w^P\right)\left(\frac{\widetilde{w}}{w^P}\right)^\beta + \widetilde{w}}{\theta W^M}.$$

Since $\frac{\partial \widetilde{\alpha}(\theta)}{\partial \theta} = -\frac{w^P\left(\frac{\widetilde{w}}{w^P}\right)^\beta - \widetilde{w}}{\theta^2 W^M} > 0$, the investor should attribute such a deviation to the highest type $\overline{\theta}$. For this belief, the investor's best response is to demand $\alpha' = \frac{K-\widetilde{w}}{\overline{\theta} W^F}$. Hence, the manager's expected payoff from deviation is strictly greater than her equilibrium payoff when deviating to $\widetilde{w} < w^P$ if her type θ' is large enough

$$\theta' > \frac{\widetilde{w}\left(1-\left(\frac{\widetilde{w}}{w^P}\right)^{\beta-1}\right)}{\left(\left(1-\frac{K-\widetilde{w}}{\overline{\theta}W^F}\right)-\left(1-\frac{K-w^P}{\overline{\theta}W^F}\right)\left(\frac{\widetilde{w}}{w^P}\right)^\beta\right)W^M}.$$

This condition is trivially satisfied when choosing $\widetilde{w} \to w^P$, implying that we cannot support a pooling equilibrium. **Q.E.D.**

Proof of Proposition 8. (i) To be able to compare debt and equity financing, we have to specify the cash flow generating process of the new project. Suppose that the good and the bad project are governed by a common cash flows generating process: $dx_t = \mu_x x_t dt + \sigma_x x_t dZ_t$ with $\mu_x, \sigma_x > 0$ and with Z denoting a Brownian motion. Let the scrap value of the project be S. Ex ante, the initial value of this process x_0 is unknown, and x_0 depends on whether the project is good or bad as well as on the realization of θ. The cumulative density function (cdf) over the possible realizations of x_0 for the good projects dominates that for the bad project in terms of FOSD. (We can assume FOSD also when comparing any two $\theta'' > \theta'$, but recall that in the baseline model θ is common knowledge.) Observe that, after the investment is sunk and the initial value has been realized, the financier cannot infer whether the realization of x_0 is due to the project being good or bad.

Before investment, the manager's and the financier's assessments of the project's expected payoff are

$$W^i(\theta) := E^i \left[\frac{x_0}{r - \mu_x} + \frac{1}{1-\beta_2} S \left(\frac{x_0}{x_d} \right)^{\beta_2} | \theta \right] \text{ where } i = F, M \; , \tag{A.21}$$

where E^M is conditional on the project being good, and E^F assumes that it is good only with probability ρ. Furthermore, β_2 is the negative root of $\frac{1}{2}\sigma_x^2 y(y-1) + \mu_x y - r = 0$ and (A.21) takes into account that the project is optimally liquidated if x_t falls below $x_d := \frac{\beta_2}{\beta_2 - 1} S(r - \mu_x)$ (see Morellec and Schürhoff (2011) for a similar derivation).[27]

Suppose now that, next to an equity share $\widetilde{\alpha}$, the manager promises the financier a small constant debt coupon payment ε and a share $\frac{\varepsilon}{r} + \widetilde{\alpha} S$ from the liquidation proceeds. Clearly, stipulating such a share is feasible for $S > 0$ and ε sufficiently small. It is now straightforward to verify that it is optimal for the manager to liquidate the project at x_d for such a liquidation sharing rule (as it is optimal for pure equity financing). The equity share $\widetilde{\alpha}$ that satisfies the financier's participation constraint is $\widetilde{\alpha} = \frac{K - w - \frac{\varepsilon}{r}}{W^F(\theta)}$. By similar arguments to Proposition 1, we obtain that before investing, the manager hoards:

$$\begin{aligned} \widetilde{w} &= \frac{\beta}{\beta - 1} \left(\left(K - W^F(\theta) \right) \frac{W^M(\theta)}{W^M(\theta) - W^F(\theta)} - \frac{\varepsilon}{r} \right) \\ &< \frac{\beta}{\beta - 1} \left(\left(K - W^F(\theta) \right) \frac{W^M(\theta)}{W^M(\theta) - W^F(\theta)} \right) = w^*. \end{aligned}$$

(ii) The payouts result is immediate. The manager's expected payoff if she pays herself a δ dividend in $t = 0$ is

$$\delta + \left(\left(1 - \frac{K - w^*}{\theta W^F} \right) \theta W^M - w^* \right) \left(\frac{w - \delta}{w^*} \right)^{\beta}$$

which is convex in δ. Hence, similarly to Proposition 6, the manager either pays out all available cash and scraps the firm or, if the growth option is sufficiently valuable, she

[27] Previously, we had $W^i(\theta) = \theta W^i$. However, recall that the results are valid as long as the assessments of firm's value $W^i(\theta)$ are increasing in θ.

doesn't pay out anything and hoards until investment. **Q.E.D.**

Proof of Proposition 9. In the first step, we complete the arguments in the main text leading to the conclusion that the manager delays less if θ is higher. In Step 2, we then show that a manager choosing private over public financing hoards less cash.

Step 1. The optimal level of effort solving (12) is

$$\begin{aligned} e^{priv} &= \frac{1}{2}m\nu\theta W \pm \frac{1}{2}\sqrt{A^{priv}} \\ \text{where } A^{priv} &= (m\nu\theta W)^2 - 4\nu\left(K - w^{priv}\right). \end{aligned} \tag{A.22}$$

Since, for any given w^{priv}, the manager's payoff is increasing in e, the global maximum is at $e^{priv} = \frac{1}{2}m\nu\theta W + \frac{1}{2}\sqrt{A^{priv}}$. This proves (13). As usual we obtain the solution for w^{priv} from the manager's first-order-condition

$$\frac{\partial U^{priv}}{\partial w^{priv}} = \left(\frac{m\nu\theta W + \sqrt{A^{priv}}}{2\sqrt{A^{priv}}}\right) - \frac{\beta}{w^{priv}}\left(\frac{\left(\frac{1}{2}m\nu\theta W + \frac{1}{2}\sqrt{A^{priv}}\right)^2}{2\nu} - w^{priv} - B\right) = 0.$$

By standard monotone comparative statics arguments, observe that $\frac{\partial^2 U^{priv}}{\partial w^{priv}\partial\theta} < 0$ implies that w^{priv} is decreasing in θ (and by analogous arguments the same holds for w^{pub}). Hence, as in our baseline model, firms with better investment opportunities hoard less cash. Analogous expressions characterize the case with public financing.

Step 2. Define $A^{pub} := (\nu\theta W)^2 - 4\nu\left(K - w^{pub}\right)$, so that the manager's effort with public financing is $e^{pub} = \frac{1}{2}\nu\theta W + \frac{1}{2}\sqrt{A^{pub}}$ in analogy to the case with private financing above. Following analogous arguments to the proof of Proposition 3, observe that when the manager is indifferent between public and private financing, it must be that

$$
\begin{aligned}
0 &= \left(\frac{\left(\frac{1}{2}m\nu\theta W + \frac{1}{2}\sqrt{A^{priv}}\right)^2}{2\nu} - w^{priv} - B\right)\left(\frac{w}{w^{priv}}\right)^{\beta} \\
&\quad - \left(\frac{\left(\frac{1}{2}\nu^{pub}\theta W + \frac{1}{2}\sqrt{A^{pub}}\right)^2}{2\nu} - w^{pub}\right)\left(\frac{w}{w^{pub}}\right)^{\beta} \\
&= \left(\left(\frac{m\nu\theta W + \sqrt{A^{priv}}}{2\sqrt{A^{priv}}}\right)\frac{\left(w^{pub}\right)^{\beta-1}}{\left(w^{priv}\right)^{\beta-1}} - \left(\frac{\nu\theta W + \sqrt{A^{pub}}}{2\sqrt{A^{pub}}}\right)\right)\frac{w^{pub}}{\beta}\left(\frac{w}{w^{pub}}\right)^{\beta} \qquad (A.23) \\
&< \left(\begin{array}{c}\left(\frac{\nu\theta W+\sqrt{(\nu\theta W)^2-4\nu(K-w^{priv})}}{2\sqrt{(\nu\theta W)^2-4\nu(K-w^{priv})}}\right)\frac{\left(w^{pub}\right)^{\beta-1}}{\left(w^{priv}\right)^{\beta-1}} \\ -\left(\frac{\nu\theta W+\sqrt{(\nu\theta W)^2-4\nu\left(K-w^{pub}\right)}}{2\sqrt{(\nu\theta W)^2-4\nu\left(K-w^{pub}\right)}}\right)\end{array}\right)\frac{w^{pub}}{\beta}\left(\frac{w}{w^{pub}}\right)^{\beta} \qquad (A.24)
\end{aligned}
$$

where the first equality (A.23) follows from the manager's first-order-conditions given private and public financing, respectively; the inequality in (A.24) follows from $m > 1$. However, for (A.24) to be positive, it must be that $w^{priv} < w^{pub}$. The analogue to part (ii) of Proposition 3 follows then the same steps as in that proposition. **Q.E.D.**

Appendix B Extensions

B.1 Cash Hoarding when Delay Reduces Uncertainty

One of the results from Section 3.2 is that firms that choose private financing delay investment less than firms choosing public financing. This result is true even if delaying investment could help alleviate the uncertainty and disagreement about the project's fundamentals. As a simple modification to our baseline model, suppose that after receiving her initial signal, the manager believes that X is X^G with probability p_0 (rather than one) and X^B otherwise. The investor disagrees, believing that the probability of X^G is $\rho < p_0$. Suppose further that before investing, the firm has a chance of observing a second signal that reveals X^G or X^B with certainty and is verifiable to all. The time until such an event follows an exponential distribution with parameter λ_e. If uncertainty disappears, the manager invests immediately if the project is good and her expected payoff is $\theta X^G - K$. If the project is bad, she abandons the project and pays out w_t. If there is no signal, the manager can choose between investing and cash hoarding as in the baseline model, but also to continue waiting to observe a signal.

The existence of such second reason to delay does not change that firms with better investment opportunities are less willing to do so. If firms use delay also for hoarding, we obtain again that firms choosing private financing delay and hoard less before investment. Hence, such firms are more likely to risk investing under uncertainty than firms choosing public financing.

Proposition B.1 *Suppose that delaying investment could alleviate uncertainty about X. Firms raising private financing are less willing to delay and are more likely to risk investing under uncertainty.*

Proof of Proposition B.1. We only show that firms with better investment opportunities experiment less. The rest of the arguments is a straightforward modification of the proofs of Propositions 2 and 3. We focus on the case in which the manager hoards cash while waiting. The alternative would be to pay out w_0, but this would have no implications for the public versus private choice. Furthermore, observe that if uncertainty unravels and $X = X^G$, public financing would be preferred, as the manager would not have to compensate the investor for bearing illiquidity costs.

Given that the true value of the investment opportunity is revealed with probability λ_e, we can use (A.15) to derive the manager's expected payoff as

$$\frac{\lambda_e p_0 \left(\theta X^G - K\right)}{(r+\lambda_e)} + \left(\left(1 - \frac{K - w^*}{\theta W^F}\right)\theta \widetilde{W}^M - w^* - \frac{\lambda_e p_0 \left(\theta X^G - K\right)}{(r+\lambda_e)}\right)\left(\frac{w_t}{w^*}\right)^{\gamma_e}$$

where γ_e is the positive root to $\frac{1}{2}\sigma^2 y\,(y-1) + \mu y - r - \lambda_e = 0$. Furthermore, we have defined $\widetilde{W}^M := p_0 X^G + (1-p_0)\, X^B$. This gives the following first-order-condition with respect to w^*

$$0 = -\frac{\gamma_e}{w^*}\left(\left(1 - \frac{K - w^*}{\theta W^F}\right)\theta \widetilde{W}^M - w^* - \frac{\lambda_e p_0 \left(\theta X^G - K\right)}{(r+\lambda_e)}\right) + \left(\frac{\theta \widetilde{W}^M}{\theta W^F} - 1\right)$$

implying that

$$w^* = \frac{\gamma_e}{(\gamma_e - 1)}\left(\frac{\left(K - \theta W^F\right)\theta \widetilde{W}^M}{\left(\theta \widetilde{W}^M - \theta W^F\right)} + \frac{\lambda_e p_0 \left(\theta X^G - K\right)\theta W^F}{(r+\lambda_e)\left(\theta W^M - \theta W^F\right)}\right).$$

Differentiating with respect to θ, we have

$$\begin{aligned}\frac{\partial w^*}{\partial \theta} &= \frac{\gamma_e}{(\gamma_e - 1)}\left(\frac{-W^F \widetilde{W}^M}{\left(\widetilde{W}^M - W^F\right)} + \frac{\lambda_e p_0 X^G W^F}{(r+\lambda_e)\left(\widetilde{W}^M - W^F\right)}\right) \\ &= -\frac{\gamma_e}{(\gamma_e - 1)}\left(p_0\left(1 - \frac{\lambda_e}{(r+\lambda_e)}\right)X^G + (1-p_0)\,X^B\right)\frac{W^F}{\left(\widetilde{W}^M - W^F\right)} < 0.\end{aligned}$$

Q.E.D.

B.2 Market Timing with Time Varying Profitability

Assuming that the NPV of the investment opportunity changes over time is a standard assumption in the related real options literature (Bolton et al., 2013). In particular, let the increase in NPV come from a lower investment outlay K. Delay in investment could, then, occur for two reasons: delaying not only to hoard cash, but also to wait for the value of the investment opportunity to increase. Our results remain robust also in such a setting.[28]

Proof: Time-varying profitability. *Time-varying profitability*: To make the argument in the simplest possible way with a closed-form solution, we make the simplifying assumption that the NPV of the project from the financiers' point of view follows

$$\frac{d\left(K - \theta W^F\right)}{\left(K - \theta W^F\right)} = \mu_K dt + \sigma_K dZ_K$$

where the change in $K - \theta W^F$ is entirely due to a change in the investment cost K. Z_K is standard Brownian motion and $\sigma_K > 0$ with a correlation ψ to Z. We assume that $\mu_K < 0$ implying that the NPV increases on average over time. Following similar steps to Proposition 1, the manager's expected payoff is the solution to the following partial differential equation

$$\begin{aligned} rU &= \mu w U_w + \frac{1}{2}\sigma^2 w^2 U_{ww} + \mu_K\left(K - \theta W^F\right)U_K \\ &\quad + \frac{1}{2}\sigma_K^2\left(K - \theta W^F\right)^2 U_{KK} + \psi\sigma\sigma_K w\left(K - \theta W^F\right)U_{wK} \end{aligned} \tag{B.1}$$

[28] Observe that even though now there are two separate reasons to delay, the firm will not necessarily delay investment longer since a lower investment outlay K implies also a lower need for hoarding cash. Furthermore, note that this insight does not depend on our assumption that the increase of NPV is due to K. The same result would obtain if the NPV would increase because of higher expected cash flows.

where the subscripts w and K denote the partials with respect to w and K, respectively. Define $\chi = \frac{w}{(K-\theta W^F)}$ so that $U(w,K) = \left(K - \theta W^F\right) U(\chi)$, where we use that U is homogenous of degree one in $(w, \left(K - \theta W^F\right))$ (A change in K that doubles $K - \theta W^F$ and doubling w would merely double the manager's expected payoff). We have

$$\begin{aligned} U_w &= U_\chi;\ U_{ww} = \frac{1}{(K-\theta W^F)} U_{\chi\chi};\ U_{wK} = -\frac{w}{(K-\theta W^F)^2} U_{\chi\chi} \\ U_K &= U - \frac{w}{(K-\theta W^F)} U_\chi;\ U_{KK} = \frac{w^2}{(K-\theta W^F)^3} U_{\chi\chi}. \end{aligned}$$

Plugging into (B.1), we obtain the simple ordinary differential equation

$$\underbrace{(r-\mu_K)}_{r'} U = \underbrace{(\mu-\mu_K)}_{\mu'} \chi U_\chi + \underbrace{\left(\frac{1}{2}\sigma^2 + \frac{1}{2}\sigma_K^2 - \psi\sigma\sigma_K\right)}_{\sigma'} \chi^2 U_{\chi\chi} \tag{B.2}$$

with a value matching condition $U(\chi^*) = \left(\frac{1-\chi^*}{\theta W^F}\right)\theta W^M - \chi^*$. Defining ϕ as the positive root to $\frac{1}{2}\sigma'^2 y(y-1) + \mu' y = r'$ (where r', μ' and σ' are defined in (B.2)), and following the same steps as in Section 3, we obtain

$$\chi^* = \frac{w^*}{(K^* - \theta W^F)} = \frac{\phi}{\phi - 1}\left(\frac{\theta W^M}{\theta W^M - \theta W^F}\right).$$

We see, thus, that the optimal co-investment w and the NPV from the financier's point of view are in a constant proportion at the optimal investment barrier. Along this barrier, the optimal cash level w^* increases with the investment cost K^*, and this level is lower when the investment opportunities are better (high θ). **Q.E.D.**

B.3 Verifying Mailath's Conditions for a Separating Equilibrium

In what follows, we verify that the regularity conditions required by Mailath (1987) are indeed satisfied and that $\frac{\partial^2}{\partial \widehat{w} \partial \theta} U(w_t, \widehat{w}, \widehat{\theta}, \theta) < 0$. Mailath's conditions are:

1) Smoothness: $U(\cdot)$ is twice continuously differentiable.

2) Belief monotonicity: $\frac{\partial}{\partial \widehat{\theta}} U(\cdot)$ is either strictly positive or strictly negative.

3) Type monotonicity: $\frac{\partial^2}{\partial \theta \partial w^{**}} U(\cdot)$ is either strictly positive or strictly negative.

4) Strict quasiconcavity: $\frac{\partial}{\partial w^{**}} U(\cdot)|_{\widehat{\theta}=\theta} = 0$ has a unique solution in w that maximizes $U(\cdot)|_{\widehat{\theta}=\theta}$, and $\frac{\partial^2}{\partial (w^{**})^2} U(\cdot)|_{\widehat{\theta}=\theta} < 0$ at this solution.

5) Boundedness: There is $k > 0$ such that for all $(\theta,\ w) \in \left[\underline{\theta}, \overline{\theta}\right] \times [0, K]$, $\frac{\partial^2}{\partial (w^{**})^2} U(\cdot)|_{\widehat{\theta}=\theta} \geq 0$ implies $\left|\frac{\partial}{\partial w^{**}} U(\cdot)_{\widehat{\theta}=\theta}\right| > k$. Note that we restrict attention to $w \in [0, K]$, as the manager

has no need of external financing if $w > K$.

Conditions 1)-2) are satisfied. Proposition 1 shows that condition 4) is also satisfied. To check for condition 5), observe that if $\frac{\partial^2}{\partial (w^{**})^2} U(\cdot)|_{\widehat{\theta}=\theta} \geq 0$, then since the first line on the RHS of

$$\begin{aligned} \frac{\partial^2}{\partial (w^{**})^2} U(w_t, w^*, \widehat{\theta}, \theta) &= \frac{\beta}{(w^{**})^2} \left(\frac{\widehat{\theta} W^F - K}{\widehat{\theta} W^F} \right) \theta W^M \left(\frac{w_t}{w^{**}} \right)^\beta \\ &- \frac{\beta}{w^{**}} \left(-\frac{\beta}{w^{**}} \left(\left(1 - \frac{K - w^*}{\widehat{\theta} W^F} \right) \theta W^M - w^{**} \right) + \frac{\theta W^M}{\widehat{\theta} W^F} - 1 \right) \left(\frac{w_t}{w^{**}} \right)^\beta \end{aligned}$$

is negative, we must have $\frac{\partial}{\partial w^{**}} U(\cdot)|_{\widehat{\theta}=\theta} < 0$. Thus, we can find a k that satisfies (5).

Finally, we check when $\frac{\partial^2}{\partial w^{**} \partial \theta} U(\cdot) < 0$ holds—i.e., condition 3). We have

$$\frac{\partial^2}{\partial w^{**} \partial \theta} U(w_t, w^*(\widehat{\theta}), \widehat{\theta}, \theta) = \left(-\frac{\beta}{w^{**}} \left(\widehat{\theta} W^F - K \right) + 1 - \beta \right) \frac{W^M}{\widehat{\theta} W^F} \left(\frac{w_t}{w^{**}} \right)^\beta .$$

A sufficient condition for this to hold is that

$$w^{**} > \underline{w} := \frac{\beta}{\beta - 1} \left(K - \underline{\theta} W^F \right). \tag{B.3}$$

Since $w^{**}(\theta)$ is lowest for type $\overline{\theta}$, it would be sufficient that $w^{**}\left(\overline{\theta}\right) \geq \underline{w}$. In this case, we could assume that observing a deviation to a lower cash hoarding strategy comes from the lowest type, making a deviation unprofitable for all types. If, instead, $w^{**}\left(\overline{\theta}\right) < \underline{w}$ and the fully separating equilibrium cannot be supported (note that condition (3) is a sufficient, but not necessary, condition), we could construct a semi-separating equilibrium in which types $\left[\theta', \overline{\theta}\right]$ pool at some $w' \geq \underline{w}$, while lower types separate with cash hoarding larger than w' in analogy to Proposition 7. It is straightforward to find out-of-equilibrium beliefs that support this equilibrium.