Preserving "Debt Capacity" or "Equity Capacity": A Dynamic Theory of Security Design under Asymmetric Information

Roman Inderst* Vladimir Vladimirov†

January 2012

Abstract

In a dynamic model of optimal security design, we show when firms should preserve "equity capacity" through choosing high target leverage or "debt capacity" through choosing low target leverage. Thereby, firms reduce a problem of underinvestment or overinvestment when they must raise future financing under asymmetric information. Which problem arises depends on whether additional financing is raised at competitive terms or whether there is a lock-in with initial investors. Firms' initial (or target) capital structure matters as it affects the "outside option" of both insiders and outside investors. Our theory also entails implications for start-up and venture capital financing.

Journal of Economic Literature Classification Codes: G32.

Keywords: Dynamic Security Design, Capital Structure, Venture Capital Financing, Asymmetric Information.

*University of Frankfurt, IMFS, and Imperial College London. E-mail: inderst@finance.uni-frankfurt.de.

†Goethe University, IMFS, and University of Amsterdam - Finance Group. E-mail: vladimirov@finance.uni-frankfurt.de.

1 Introduction

Successful firms rarely raise financing only once in their liftetime. The key contribution of this paper is to recognize that when they intend to raise fresh financing, firms' existing capital structure shapes the "outside options" of both inside owners and outside investors, i.e., the payoffs that they can expect when *no* new financing is raised. This link through the respective "outside options" ensures that a young firm's initial capital structure or the target capital structure of a mature firm matter when the firm must raise additional financing, possibly at short notice, under asymmetric information.

While the notion that firms' financial structure is shaped by problems of *asymmetric information* has a long-standing tradition in the theory of corporate finance (Myers and Majluf, 1984), our novel contribution is to embed the most standard problem of asymmetric information into a simple, two-period dynamic model and, thereby, obtain a rich set of implications from a single agency problem. We show when the "pecking order" theory should hold or when it should fail to hold, and we show when firms should optimally preserve "debt capacity" and when they should rather preserve "equity capacity" in order to improve efficiency.

Firms that expect to be locked-in with investors who exert substantial bargaining power at later financing rounds benefit from initially preserving *equity capacity* so as to reduce an *underinvestment* problem in the future. This allows to make it more attractive for investors to finance future growth. Instead, firms that expect to receive future financing at competitive terms are better off preserving *debt capacity* so as to reduce an *overinvestment* problem in the future. In this case, debt is used to reassure investors that only sufficiently profitable opportunities are financed.

Our results speak to the question how firms should choose a target capital structure, from which they then deviate when financing is raised (at short notice) under asymmetric information. By taking our two-period model more literally, we also relate our results to the literature on venture capital and small-firm financing. We derive conditions for when firms optimally have low leverage initially, but subsequently increase leverage as they raise additional financing. We interpret our results also in terms of the use of convertible securities. As we obtain quite different predictions for the optimal choice of leverage and its change over time, depending on whether there is an over- or an underinvestment problem, our results help to organize the somewhat contradicting recent evidence. For instance, we confirm the "pecking order" when additional capital can be raised at competitive terms. If, instead, refinancing conditions are determined by a relationship investor with strong bargaining power, we predict that leverage decreases when new financing must be raised under asymmetric information.

The case in which firms can always raise (additional) financing at competitive terms is most closely related to the extant theoretical literature. Then, refinancing under asymmetric information will increase leverage, which confirms the predictions of the "pecking order" (Myers and Majluf, 1984; Nachman and Noe, 1994). Debt financing limits *overinvestment*. In fact, in our dynamic model, where the firm has an endogenously chosen initial capital structure in place, we show that debt financing may even ensure that investment is first-best efficient. The firm's optimal initial (or target) financial structure maximally preserves the firm's "debt capacity". This increases efficiency when future financing is needed. Firms with growth opportunities and access to competitive financing should thus have a low target leverage ratio and they should temporarily increase leverage when fresh outside funding is needed. These predictions are overturned when firms are "locked-in" with existing (relationship) investors, at least when future financing has to be raised under asymmetric information, e.g., at short notice. With a strong investor, refinancing leads to a reduction, rather than to an increase in leverage.

These richer predictions of our model are in line with recent evidence that does not universally support the pecking order (Fama and French, 2005; Leary and Roberts, 2010). One reason why incumbent investors may have bargaining power, which is when the pecking order fails, is that their refusal to (co-)finance new investment may make it impossible for the firm to raise new financing elsewhere. Such a refusal sends a negative signal about the firm's overall prospects ("lemons"). For this case, we show that underinvestment can result as investors seek to extract a larger share of the gains from a new investment. There is less underinvestment if refinancing leads to a decrease in the firm's leverage. The optimal initial capital structure should exhibit high leverage, as it preserves the firm's "equity capacity" and, thereby, mitigates the risk of future underinvestment.

We extend the most simple model of security design under asymmetric information in a natural way, namely by making also the initial capital structure endogenous. When we interpret the (initial) capital structure under symmetric information as firms' long-term choice, we obtain predictions both for firms' target capital structure and for the direction of deviations. The key distinction is whether firms can raise financing competitively even under asymmetric information, as this determines whether there will be a problem of over-investment or underinvestment. The initial (or target) capital structure affects efficiency as it shapes the "outside options" of both insiders and initial outside investors. In terms of predictions, we argue that (large) firms that can raise capital at competitive terms also under asymmetric information will do so by issuing debt, though their target capital structure involves little debt, but small firms facing a specialized or relationship investor will issue equity when they need additional financing, though their target capital structure involves high leverage. These predictions are in line with findings by Shyam-Sunder and Myers (1999) or Frank and Goyal (2003). It has also been documented that equity is often raised by firms that are not under duress (Fama and French, 2005), possibly so as to thereby stock-pile debt capacity (Lemmon and Zender, 2009). In contrast, smaller growth firms have been shown to issue debt when asymmetric information is *not* a factor, so as to preserve equity capacity for the long run (Leary and Roberts, 2010; Gomes and Phillips, 2005). We discuss below in more detail our implications and how they compare to existing findings.

Our different results, depending on whether there is a problem of under- or overinvestment when financing must be raised under asymmetric information, also help to organize some recent conflicting evidence on start-up and young firm financing. For this interpretation we take our two-stage model of financing more literally. The optimal choice of securities over time then depends crucially on outside investor versus owner-manager bargaining power.[1] Such a difference in bargaining power may, for instance, arise from differences in the legal environment (e.g., Lerner and Schoar, 2005; Cumming 2005, 2008; Kaplan et al., 2007). We show that if bargaining power is in the hands of the owner-manager at the refinancing stage, then it is optimal to initially finance with junior securities and to switch to more senior securities at later stages. In accordance with this prediction, the above studies find that in countries with weak creditor rights or weak enforcement of these rights equity is more frequently issued in first rounds and for younger companies. More successful firms then issue more senior securities in later rounds of financing (in particular, Kaplan

[1] With regards to bank lending, see Degryse and Ongena (2008) for a review of the empirical evidence on "rent-extraction" over the duration of a bank-borrower relationship.

et al., 2007). In contrast, if firms face a strong investor, our results prescribe the opposite behavior, and firms should then initially raise finance through issuing senior claims. In line with this result, start-up firms in countries with a strong protection of creditor rights and strong law enforcement issue securities that initially give the investor downside protection, but which may be converted to more junior claims with a higher upside participation during expansion in later financing rounds (e.g., Kaplan and Strömberg, 2003).

In terms of corporate finance theory, a novel contribution of our model is to solve for a security design problem under asymmetric information where both the privately informed owner-manager and the original investors already have a stake in the company. Their existing claims create "outside options", whose value depends on the firm's profitability when no new financing is raised. Technically, we thus solve for a game of screening (when the investor has bargaining power) and a game of signaling (when the owner-manager has bargaining power) with so-called type-dependent reservation values.[2] By affecting the "outside options" at the refinancing stage, the firm's initial financing structure becomes relevant even though it is chosen under symmetric information. We obtain conditions when despite private information at the refinancing stage, the outcome is efficient, i.e., there is no under- or overinvestment. This is made possible as also the value of the stakeholders' "outside options" depends on the firm's profitability. Since in practice (fresh) financing is frequently raised when the firm already has outstanding securities, our two-period model of financing under asymmetric information should add realism.[3]

There is a growing body of research on the dynamics of a firm's optimal capital structure, trading off tax benefits, bankruptcy and agency costs, or the occurrence of technology shocks, for instance, (cf., Hennesy and Whited, 2005; Miao, 2005). In contrast to this literature, we derive our results from a single inefficiency, namely interim private information, in a stylized two-period setting. In a related paper, Axelson et al. (2009) also take a security design approach in a model with interim private information. Similarly to our case of a "weak" investor at the refinancing stage, they derive debt as the unique interim security. This confirms the predictions of Myers and Majluf's (1984) pecking order theory (cf., also Nachman and Noe, 1994). In our model, however, (re)financing is not always

[2] As we argue below, however, a single security instead of a menu is always optimal at the refinancing stage.

[3] In different contexts, the role of (type-dependent) outside options has recently been explored, for instance, by Tirole (2011) and Burkart and Lee (2011).

optimal for the owner-manager, as the existing capital structure serves as an outside option. The overinvestment problem is then less severe. In fact, as we noted above, first-best investment may be achieved, and the pecking order is reversed if the bargaining power shifts to the uninformed investor.[4] Though DeMarzo and Duffie (1999) and Biais and Mariotti (2005) also consider a two-stage game, the security in their models is designed before private information is revealed, and ultimately only a single security is issued.

The separation of initial financing and possible refinancing relates our paper also to the literature on stage financing, which is a well documented fact in start-up finance (e.g., Gompers, 1995; Kaplan and Strömberg, 2003). A number of authors have used staging in a security design context when there is no commitment to refinancing (e.g., Cornelli and Yosha, 2003). Further, following Aghion and Bolton (1992), several papers have motivated the use of contingent securities in venture capital financing in the context of incomplete contracting (e.g., Berglöf, 1994). We add to this literature by obtaining unique and contrasting optimal securities from an interim adverse selection problem. Our contribution lies especially in showing how variations in bargaining power, as arising from different legal environments or different stages of firm development, significantly change optimal security design.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 examines the optimal initial and interim financing when the investor has bargaining power at the refinancing stage. Section 4 characterizes security design when the owner-manager has bargaining power at the interim stage. Empirical implications of our results are presented in Section 5, and Section 6 concludes. All proofs are in the Appendix.

2 The Model

We envisage a firm that raises financing in an initial period, $t = 1$, and that raises additional financing in a subsequent period, $t = 2$. Precisely, suppose that at $t = 1$ a penniless owner-manager ("she") needs to raise initial financing $K_1 > 0$. At $t = 2$, another investment

[4] The optimal ex ante financing in our model, which is chosen under symmetric information, is also very different from Axelson et al. (2009). They propose a mixture of debt and levered equity: Debt is used to deter the entry of fraudulent entrepreneurs, while levered equity mitigates the risk shifting incentives caused by debt financing. In one of our cases we also obtain that levered equity may be optimal, because it reduces overinvestment in later stages. What drives this result, however, is the role of levered equity for outside options in the refinancing negotiations.

$K_2 > 0$ can be made. We argue below why it is not possible to raise all financing initially. For simplicity only, we assume that the investment opportunity in $t = 2$ arises with probability one. The investment opportunity will be profitable only sometimes, though. Cash flows are realized in the final period, $t = 3$. Both the owner-manager and investors are risk neutral, and we abstract from discounting. We next add more details to our model.

Financing and Contracting The firm's verifiable cash flow at $t = 3$ is either low or high: $x_l \geq 0$ or $x_h > x_l$. The likelihood of realizing high cash flow depends both on whether additional capital was injected at $t = 2$ and on the firm's underlying profitability (its "type"). This is made precise when discussing the information structure below. Our restriction to only two cash flows is only for convenience, as results can be generalized to a setting with a continuous cash-flow distribution (cf. on the respective generalization of assumptions footnote 6 below).

To raise K_1 at $t = 1$, the owner–manager issues a security $R^1(x)$ that conditions the repayment on the final cash flow. The firm can initially raise capital competitively, so that the resulting security design problem will be to maximize the ex-ante value of the owner-manager's claims. For this reason, we stipulate that at $t = 1$ the owner-manager can offer $R^1(x)$ to investors.

Raising financing at $t = 2$, provided this is successful, involves a fresh injection of K_2 by the investor. Then, the initial security $R^1(x)$ that is held by outside investors is replaced by a new security $R^2(x)$. Here, it is convenient to suppose that all outside claims are held by one investor. However, as we argue below our results are applicable more broadly, so that there could be, for instance, fresh financing from new investors. But we also discuss applications where it is reasonable that there is a single investor, e.g., a housebank or a venture capital investor. We also discuss below how our two stages, $t = 1$ and $t = 2$, can be interpreted as representing a firm's long-term and short-term choice of financing and leverage.

We make the standard assumptions that $0 \leq R^t(x) \leq x$ and that both $R^t(x)$ and $x - R^t(x)$ are nondecreasing. According to the first assumption, the security can only distribute the cash flows that are realized by the firm. As the owner-manager is assumed to be penniless, the relevant restriction is that $R^t(x) \geq 0$: The security can not specify a "wage" that is paid to the owner-manager over and above the firm's cash flow. This assumption is common in the literature. Such a payment could lure "non-serious" operators into the market. Also, it is common to assume that both $R^t(x)$ and $x - R^t(x)$, i.e., the payouts to outside investors and the owner-manager, are nondecreasing. Otherwise, either party could have an incentive to "destroy" cash flow by obstructing the operations of the firm.

Information The firm's profitability can take on two values. We refer to these as a good or bad "state", $\theta = \{B, G\}$. A priori, the likelihood of $\theta = G$ is given by $0 < \widehat{q} < 1$. At $t = 1$, this is common knowledge between the owner-manager and potential investors. At $t = 2$, when the refinancing decision must be made, the owner-manager has already gained an informational advantage regarding θ, which she cannot credibly share with the investor, at least not at such short notice. Based on this private information, the posterior belief of the owner-manager regarding $\Pr(\theta = G)$ is q. We refer to the owner-manager's private information q as her "type" at $t = 2$. It is a priori distributed according to some CDF $F(q)$ over $q \in [0, 1]$, satisfying $\widehat{q} = \int q dF(q)$.[5]

Together with the decision whether to refinance at $t = 2$, $d \in D := \{Y, N\}$ ("Yes" and "No"), the firm's profitability $\theta = \{B, G\}$ determines the probability of high cash flows: $p_{d\theta}$. Assuming that $p_{dG} > p_{dB}$ holds for all $d \in D$, $\theta = G$ can be unambiguously referred to as the "good" sate irrespective of whether refinancing was obtained or not. Further, refinancing has a larger impact on the firm's upside potential if the firm's profitability is $\theta = G$ rather than $\theta = B$, i.e., if the firm's prospects are generally more attractive:

$$p_{YG}/p_{NG} > p_{YB}/p_{NB}. \tag{1}$$

We discuss the importance of condition (1) for our characterization of unique optimal securities below. Note that it implies that $p_{YG} - p_{NG} > p_{YB} - p_{NB}$.[6]

[5] Alternatively, we may, instead, stipulate that the owner-manager privately observes some signal ϑ, which is generated by the CDFs $\Psi_\theta(\vartheta)$. We can then generate q as well as $F(q)$ by using Bayes' rule.

[6] As noted above, our results can be generalized to a setting with a continuous cash-flow distribution. Formally, with a CDF $H_d(x|\theta)$ for all combinations $d = \{Y, N\}$ and $\theta = \{G, B\}$ we can first generalize $p_{d\theta}(x) := 1 - H_d(x|\theta)$. Assume that the distribution for G dominates that for B in terms of conditional stochastic dominance (CSD): $p_{dG}(x'|z) \geq p_{dB}(x'|z)$ for $x', z \in X$, where $p_{d\theta}(x|z)$ is the conditional probability $1 - \Pr(x' \leq x \leq x' + z)$. This implies that high cash flows are increasingly more probably in state G compared to state B: $\frac{\partial}{\partial x}\left(\frac{p_{dG}(x)}{p_{dB}(x)}\right) \geq 0$. Further, to generalize condition (1), we would require that $\frac{p_{YG}(x)}{p_{YB}(x)} \geq \frac{p_{NG}(x)}{p_{NB}(x)}$ holds for all x. We have shown in a working-paper version how these assumptions jointly ensure that all our subsequent results on optimal security design extend to a setting with a continuous cash-flow distribution.

Discussion of Contracting For our analysis, we distinguish two cases for the game that proceeds at the refinancing stage $t = 2$. We capture the case where the firm is "locked-in" with the initial investor by granting the investor all bargaining power: It is the investor who makes a take-it-or-leave-it offer. When there is no such "lock-in", the owner-manager has all bargaining power. Then, it is the owner-manager who makes a take-it-or-leave-it offer. While these two settings are certainly extreme, they allow to capture the fundamental sources of inefficiency that arise from asymmetric information.

After deriving our first characterization, we will be more explicit about endogenizing a possible "lock-in", e.g., through an additional layer of information asymmetry between initial investors and new investors at $t = 2$. Then, we will also motivate our assumption that not all of the required funds, $K_1 + K_2$, are raised initially and that we can indeed restrict attention to a sequence of simple contractual games: raising initial financing in $t = 1$ through $R^1(x)$ and refinancing in $t = 2$ through an exchange for $R^2(x)$.

Additional Notation and Parameter Restriction To ease exposition, we use the following short-hand notation: $R_l^t := R^t(x_l)$ denotes the repayment for low cash flow, $\Delta R^t := R^t(x_h) - R^t(x_l)$ denotes the outside investor's upside, and $\Delta x := x_h - x_l$ denotes the upside of the whole firm. One can then represent the investor's payoff when there is refinancing at $t = 2$ as

$$v_Y(R^2, q) := R_l^2 + (p_{YB} + q\,(p_{YG} - p_{YB}))\,\Delta R^2 - K_2,$$

depending on the refinancing security contract R^2 and the owner-manager's private information (his "type") q. Note that the payoff is gross of the initial outlay K_1, but net of K_2. Likewise, denote the investor's expected payoff without refinancing by

$$v_N(R^1, q) := R_l^1 + (p_{NB} + q\,(p_{NG} - p_{NB}))\,\Delta R^1.$$

Total expected cash flow in either case is given by the joint surplus

$$\begin{aligned} s_Y(q) \;&:\; = x_l + (p_{YB} + q\,(p_{YG} - p_{YB}))\,\Delta x - K_2, \\ s_N(q) \;&:\; = x_l + (p_{NB} + q\,(p_{NG} - p_{NB}))\,\Delta x. \end{aligned}$$

With this at hands, the owner-manager's expected payoff can be written as

$$\begin{aligned} u_N(R^1, q) \;&:\; = s_N(q) - v_N(R^1, q), \\ u_Y(R^2, q) \;&:\; = s_Y(q) - v_Y(R^2, q). \end{aligned}$$

To limit trivial case distinctions, we suppose that refinancing is efficient only in $\theta = G$:

$$(p_{YB} - p_{NB})\Delta x < K_2 < (p_{YG} - p_{NG})\Delta x. \tag{2}$$

This implies that there exists a cutoff $0 < q_{FB} < 1$, so that refinancing increases the joint surplus only if the owner-manager's type (the probability of being in G) is above q_{FB}.

Similarly, it is convenient to stipulate that

$$K_1 > x_l,$$

as this allows to rule out the use of safe debt. Throughout the analysis we assume that the firm is financially viable at $t = 1$ under the respective optimal contracts.[7]

3 The Case with a Strong Investor

We proceed backwards and take first the refinancing stage at $t = 2$. The equilibrium outcome is then plugged into the problem that arises at the initial financing stage, $t = 1$. Recall that we presently consider a game where the investor can make a take-it-or-leave-it offer to the owner-manager at $t = 2$. Hence, while initially the firm faces a competitive market for capital, the initial investor has bargaining power vis-à-vis the (locked-in) firm when it needs refinancing at $t = 2$.

3.1 Refinancing (Strong Investor)

To obtain refinancing K_2, the owner-manager must agree to replace the existing security R^1 by R^2, as proposed by the investor. Otherwise, no new capital K_2 is injected and the original security R^1 stays in place.

Denote the set of all types q who accept the offer by $q \in A \subseteq [0, 1]$. That is, $u_Y(R^2, q) \geq u_N(R^1, q)$ holds for all $q \in A$, and $u_Y(R^2, q) < u_N(R^1, q)$ otherwise.[8] Then, the investor's expected payoff at $t = 2$ is given by

$$\int_A v_Y(R^2, q)dF(q) + \int_{[0,1]/A} v_N\left(R^1, q\right)dF(q). \tag{3}$$

The investor's objective is thus to choose R^2 so as to maximize (3), subject to the imposed "feasibility" constraints for R^2: $0 \leq R^2(x) \leq x$ and that both $R^2(x)$ and $x - R^2(x)$ are non-decreasing. This program is solved next.

[7] After deriving the optimal contracts, a condition that is both necessary and sufficient can be obtained, albeit only implicitly in terms of the primitives.

[8] Resolving the owner-manager's indifference in this way will be inconsequential for the further analysis.

Optimal Refinancing Security Arguably, the best that the investor can do is to ensure that the efficient decision is made while extracting the full surplus from refinancing. We show that this may be feasible even though the owner-manager has private information about the benefits from refinancing. This requires that each owner-manager type $q \in A$ is exactly indifferent between accepting and rejecting the investor's offer:

$$u_Y(R^2, q) = u_N(R^1, q) \; \forall q \in [0, 1], \tag{4}$$

so that for the incremental payoff of the investor it indeed holds that

$$v_Y(R^2, q) - v_N(R^1, q) = s_Y(q) - s_N(q).$$

Note that this is just a formal restatement of the requirement that the investor extracts the whole NPV from the new investment. Note that for the owner-manager it is from (4) indeed optimal to choose the acceptance set $A = [q_{FB}, 1]$. Resolving his indifference (for all q) in this way is, however, not an assumption: It clearly must apply in equilibrium, provided that such a contract R^2 is feasible, as otherwise the investor could marginally adjust the contract to break the manager's indifference to the left and the right of $q = q_{FB}$.

We now construct the respective security so that (4) is satisfied, which we denote by $R^2 = \widehat{R}$. As it implements the first-best acceptance decision, we also refer to it as the "first-best" security, though it should be recalled that it also allows the investor to extract all incremental surplus from refinancing. For any initial security R^1, one can solve (4) to obtain the "upside" $\Delta\widehat{R}$ and the respective safe repayment, $\widehat{R}_l$:

$$\begin{aligned} \Delta\widehat{R} &= \Delta x - \frac{p_{NG} - p_{NB}}{p_{YG} - p_{YB}} \left(\Delta x - \Delta R^1\right), && (5) \\ \widehat{R}_l &= R_l^1 - p_{NB} \left(\Delta x - \Delta R^1\right) + p_{YB} \left(\Delta x - \Delta\widehat{R}\right). && (6) \end{aligned}$$

We first comment on the properties of this contract before discussing when it is feasible. The characterization in (5), together with condition (1), imply that $\Delta\widehat{R} > \Delta R^1$, while from (6) it then follows that $\widehat{R}_l < R_l^1$. Hence, the "first-best" security $\widehat{R}$ offered in $t = 2$ is unambiguously steeper from the investor's perspective than the initial security R^1. The intuition is immediate. Recall that refinancing has a larger impact on expected cash flows when the firm is more profitable, $\theta = G$, compared to when $\theta = B$ (cf. condition (1)). To keep *all* types q indifferent between the old security and the new security with additional financing, as required by (4), the owner-manager must receive less from the upside and more from the safe cash flow under security R^2, compared to the initial security R^1. Only then will she be indifferent between refinancing and non-refinancing for all q. Consequently, the claims held by the outside investor, $R^2 = \widehat{R}$, are strictly steeper than under the initial security R^1.

The "first-best" security, $\widehat{R}$, may not be feasible, though. This is the case when the new security can not be made sufficiently steep, as $\widehat{R}$ would require that $\widehat{R}_l < 0$, which is not feasible. That is, in order to indeed extract the full surplus from the new investment, the investor would have to demand a security with a "negative repayment" $\widehat{R}_l < 0$.[9] The optimal *feasible* security will then be flatter than $\widehat{R}$, and not all of the surplus from refinancing can be extracted by the investor. That is, the difference in the owner-manager's expected payoff with and without refinancing, $u_Y(R^2, q) - u_N(R^1, q)$, is no longer zero everywhere, as it is when $R^2 = \widehat{R}$, but it is strictly increasing in q. For this case, denote the unique point of intersection of $u_Y(R^2, q)$ and $u_N(R^1, q)$ by q^*:[10]

$$u_Y\left(R^2, q^*\right) = u_N\left(R^1, q^*\right). \tag{7}$$

The set of owner-manager types who accept the refinancing offer in $t = 2$ becomes thus $A = [q^*, 1]$: The owner-manager prefers to accept R^2 if $q \geq q^*$ and strictly so if $q > q^*$, while she prefers to reject the new offer if $q < q^*$. All types $q > q^*$ now receive an *information rent* of size

$$u_Y(R^2, q) - u_N\left(R^1, q\right). \tag{8}$$

For each type $q > q^*$, we can say that the rent (8) is a payoff that the investor loses, as he presently can exert his bargaining power when proposing the security R^2. Consequently, the investor wants to make the expected rent as small as possible. Note that for each $q > q^*$, the rent (8) is arguably smaller when the owner-manager's expected payoff under refinancing becomes flatter. To make $u_Y(R^2, q)$ flatter, i.e., to decrease the slope everywhere, the owner-manager's residual claim $x - R^2(x)$ must become flatter, which in turn requires that the investor's claim R^2 becomes steeper. Hence, if $\widehat{R}$ is not feasible, the best the investor can do is to make the new security R^2 as steep as possible, through

[9] As noted above, this could prescribe a wage that is paid to the owner-manager regardless of the firm's performance, which - following standard restrictions - we excluded.

[10] While we could simply set $q^* = 0$ for the case where there is no intersection as also $u_Y\left(R^2, q = 0\right) > u_N\left(R^1, q = 0\right)$, this case will not arise by optimality for the investor. In fact, we show that always $q^* \geq q_{FB}$.

maximizing ΔR^2, while ensuring that R^2 remains feasible.

Proposition 1 *Suppose that there is a strong investor, who makes the offer at the refinancing stage. Then, when refinancing is successful, the investor holds a security R^2 that is steeper than the initial security R^1: $R_l^2 \leq R_l^1$ and $\Delta R^2 \geq \Delta R^1$ (the inequalities are strict if initially $R_l^1 > 0$ or $\Delta R^1 < \Delta x$). We further have the following case distinction:*
(i) If

$$R_l^1 \geq \left(\frac{p_{YG} p_{NB} - p_{YB} p_{NG}}{p_{YG} - p_{YB}} \right) \left(\Delta x - \Delta R^1 \right) \tag{9}$$

holds, then the "first-best" security $R^2 = \widehat{R}$, as characterized in (5)-(6), is feasible and uniquely optimal, in which case the refinancing decision is always efficient: There is refinancing if and only if $q \geq q^* = q_{FB}$.
(ii) Otherwise, i.e., if (9) does not hold, the new security is levered equity with $R_l^2 = 0$, and there is refinancing if and only if $q \geq q^*$. There is underinvestment as $q_{FB} < q^* < 1$.

Proof. See Appendix.

We postpone the discussion of condition (9), as we presently take the initial security R^1 as given. Hence, we return to this condition once we have solved also for the optimal initial security R^1. If condition (9) does not hold, Proposition 1 pins down levered equity with $R_l^2 = 0$ as the uniquely optimal *shape* of the optimal security at the refinancing stage. This maximally shifts the claim of the owner-manager to the low cash-flow realization and that of the investor to the high cash-flow realization. By thereby making the investor's payoff from refinancing as steep as possible and that of the owner-manager as flat as possible, the owner-manager's information rent is minimized. Note that we presently take a general security design perspective and thus consider the full replacement of the initial security R^1 by a new security R^2. After deriving the optimal initial security R^1 in the following section, we offer more interpretation for Proposition 1, e.g., in terms of a change in leverage or in terms of converting an existing security.

Rephrasing the Investor's Trade-Off If $\widehat{R}$ is not feasible (cf. case ii) in Proposition 1), the investor faces a trade-off between maximizing surplus and reducing the information rent that the owner-manager can extract. The solution of this trade-off results in inefficient refinancing: $q^* > q_{FB}$, so that there is underinvestment at the refinancing stage. It remains to pin down the resulting cutoff q^*. For this we can now substitute the cutoff rule, i.e., $A = [q^*, 1]$, into the investor's objective function (3). Further, in case of inefficient refinancing, we can substitute $R_l^2 = 0$, as we know that levered equity is the uniquely optimal refinancing security. We are thus left with a single remaining contracting variable to solve for, ΔR^2. From the owner-manager's indifference condition for q^* (cf. condition (7)) we obtain the upside of the investor's repayment ΔR^2 as a monotonic function of q^*. Implicitly differentiating (7) yields $d\Delta R^2/dq^* > 0$: As ΔR^2 increases, the investor extracts more from the upside and the owner-manager is left with a smaller share, so that only higher types of the owner-manager still find it optimal to accept the refinancing offer, which pushes up q^*. Taken together, we can thus differentiate the investor's profits (3) with respect to q^* to obtain the first-order condition

$$-\left[s_Y(q^*) - s_N(q^*)\right] f\left(q^*\right) + \frac{d\Delta R^2}{dq^*} \int_{q^*}^{1} \frac{dv_Y(R^2, q)}{d\Delta R^2} dF\left(q\right) = 0. \tag{10}$$

(For a more explicit derivation, including $d\Delta R^2/dq^*$, see the proof of Proposition 1). The second term in (10) captures the benefits from "implementing" a higher q^* and, thereby, extracting a higher payoff from all types $q > q^*$. The respective loss in surplus, given that the underinvestment problem becomes more severe, is captured by the first term in (10).

Discussion If the incumbent investor makes the refinancing offer, as an expression of his bargaining power, the outcome can be inefficient, as there may be underinvestment. The investor's offer then trades off joint surplus maximization with information rent extraction. This rent for the owner-manager arises as she is privately informed about the firm's profitability θ. As discussed above, this can capture the notion that refinancing must be raised at sufficiently short notice.

Note that while we restrict the investor to making a single offer, R^2, this is without loss of generality, as it is straightforward to show that the investor would not gain from offering a menu of securities. Intuitively, any non-degenerate menu of contracts would have to include also a flatter security, which the owner-manager would choose for lower values q, thereby realizing more instead of less information rent.

As we noted above, the fact that the owner-manager's profit without refinancing also depends on the type q ("type-dependent outside option") may even allow the investor to extract all surplus. This leaves the owner-manager with zero information rent, making the outcome efficient. This interaction between the initial security R^1 and the efficiency of the refinancing decision under the investor's optimal offer R^2 is explored next, as we solve for the initial stage $t = 1$.

3.2 Raising Initial Finance (Strong Investor)

Initially, at $t = 1$, there is no private information, and financing can be raised at competitive terms. We model this by granting the owner-manager the right to make the offer to the investor. Using that $u_d(R^t, q) = s_d(q) - v_d(R^t, q)$, she maximizes her expected payoff

$$\int_0^{q^*} \left(s_N(q) - v_N\left(R^1, q\right)\right) dF(q) + \int_{q^*}^1 \left(s_Y(q) - v_Y\left(R^2, q\right)\right) dF(q) \tag{11}$$

subject to the participation constraint of the investor at $t = 1$

$$\int_0^{q^*} v_N\left(R^1, q\right) dF(q) + \int_{q^*}^1 v_Y\left(R^2, q\right) dF(q) \geq K_1, \tag{12}$$

where, importantly, q^* and R^2 are determined at the interim stage. Note that, to simplify the exposition, we have presently assumed that, for given R^1, the investor chooses a pure strategy in $t = 2$, so that R^2 and q^* are pinned down uniquely. As we show in the proof of Proposition 2, in analogy to a tie-breaking condition, this must indeed hold in equilibrium, even though once we arrive at $t = 2$, the investor's program may not be strictly quasiconcave.

Suppose first that there is inefficiency at the interim stage irrespective of the security contract that was offered initially. Below we present a simple condition when this is the case.

Proposition 2 *Take the case with a strong investor who determines the refinancing terms at $t = 2$. Then, if there is underinvestment at $t = 2$ ($q^* > q_{FB}$), it is uniquely optimal for the firm to raise initial financing at $t = 1$ through a debt contract, $R_l^1 = x_l$.*

Proof. See Appendix.

Debt is the flattest security from the investor's perspective and thus the steepest security from the owner-manager's perspective. Recall that if there is inefficiency at $t = 2$ (cf. the first-order condition (10)), it is due to the fact that the investor wants to extract more of the owner-manager's information rent. If the initial claim R^1 becomes flatter, the owner-manager's residual claim and thus also her expected payoff without financing (her "outside option") become steeper in q. This reduces her information rent, $u_Y\left(R^2,q\right)-u_N\left(R^1,q\right)>0$ for all $q>q^*$, where we know from Proposition 1 that R^2 is levered equity. As the owner-manager's information rent is reduced, it has less weight in the maximization problem of the investor at $t=2$. Therefore, he resolves the trade-off between rent extraction and efficiency by making a refinancing offer that leads to a lower, more efficient q^*. Put differently, raising the initial amount K_1 through debt financing preserves the firm's upside potential - its "equity capacity" - for the latter round of financing. The potential to replace the initial security by a *relatively* steeper ("levered equity") security allows then to reduce underinvestment.

What matters is thus how steep the new security in case of refinancing, R^2, can become *relative* to the initial security R^1. It is through this channel that the initial financing decision affects efficiency at the refinancing stage. Even though initial financing is chosen under symmetric information (and absent any other agency problem, for that matter), Proposition 2 still pins down a unique security. Debt preserves the maximum "equity capacity" to alleviate the underinvestment problem in $t=2$.

Observe now from condition (9) that an efficient outcome in $t=2$ is feasible only if R_l^1 is sufficiently high. Only then can the new security $R^2=\widehat{R}$ be made sufficiently steep *relative* to R^1. Denote the maximum feasible joint surplus, after subtraction of the initial outlay K_1, by

$$S_{FB}:=\int_0^{q_{FB}}s_N\left(q\right)dF\left(q\right)+\int_{q_{FB}}^1 s_Y\left(q\right)dF\left(q\right)-K_1$$

and, to ease notation for the rest of the paper, let

$$p_d\left(q\right):=p_{dB}+q\left(p_{dG}-p_{dB}\right)\ \text{ for } d=\{Y,N\}\,. \tag{13}$$

We derive now a simple condition showing when it is feasible to construct an initial security that leads the investor to implement the efficient cutoff $q^*=q_{FB}$ and when, instead, it holds at $t=2$ that $q^*>q_{FB}$.

Proposition 3 *If a strong investor determines the refinancing conditions in $t=2$, the first-best investment outcome ($q^*=q_{FB}$) is obtained if*

$$x_l\geq\frac{p_{NB}p_{YG}-p_{YB}p_{NG}}{(p_{YG}-p_{YB})p_N(\widehat{q})}S_{FB}+\frac{(p_{NG}-p_{NB})p_Y(\widehat{q})}{(p_{YG}-p_{YB})p_N(\widehat{q})}\max\left(0,S_{FB}-p_N(\widehat{q})\Delta x\right), \tag{14}$$

while otherwise there is underinvestment with $q^* > q_{FB}$. In both cases, the security that is held by the investor after refinancing is unambiguously steeper from the investor's perspective than the initial security ($\Delta R^2 > \Delta R^1$ and $R_l^2 < R_l^1$).

Proof. See Appendix.

Condition (14) is intuitive in light of the previous discussion. If x_l is large enough, the owner-manager can ensure that the investor just breaks even with a security that is sufficiently flat for the investor so that, at $t = 2$, the "first-best" security is feasible. Also in this case refinancing will lead to a strictly steeper security for the investor. However, if condition (14) holds strictly, the initial security R^1 is not pinned down uniquely. In this case there is some leeway in choosing the optimal security R^1 while still realizing the efficient outcome.

Incidentally, if (14) does not hold, so that there will be underinvestment in equilibrium, the owner-manager could still offer an initial security that would subsequently induce first-best refinancing with $q^* = q_{FB}$, albeit then the investor would realize strictly more than what is required to make him break even. That is, the investor's ex-ante expected payment would strictly exceed his expected capital outlay. (Trivially, this would always be the case if the owner-manager no longer had a stake in the firm as $R^1 = R^2 = x$.) This is, however, never optimal. Intuitively, at $q^* = q_{FB}$ a marginal distortion has a zero first-order effect on total surplus, while a reduction in the investor's ex-ante payoff has a first-order effect on the owner-manager's payoff.[11]

3.3 Discussion

Taken together, Propositions 2 and 3 make the following assertions. If refinancing is obtained from a strong investor, e.g., an investor with whom the firm is presently "locked-in", then the claims held by outside investors should become *steeper* after refinancing. There is scope for underinvestment, especially for firms that repay little in case of failure, relative to total profits (when condition (14) does not hold). Somewhat loosely speaking, such a problem of underinvestment at the refinancing stage should thus be expected more

[11] Note that we do not use for our arguments that under the optimal initial security, R^1, the investor just breaks even. Debt is uniquely optimal in Proposition 2 as it allows to reduce underinvestment in $t = 2$ for any *given* level of the investor's ex-ante payoff. The efficiency gains obtained thereby accrue to the owner-manager.

for firms with a more severe downside risk. To reduce this inefficiency, the firm will initially raise financing through debt, thereby choosing high leverage. When refinancing is obtained, the change in outside investors' claims reduces leverage. Precisely, for the case with underinvestment, we obtain that outside financing turns into levered equity at the refinancing stage. In Section 5, after we have characterized the outcome also for the case with a weak investor, we offer various interpretations for these results and relate them to existing evidence on firms' choice of financing.

Assumptions and Interpretation An important feature of our model, which is shared with many of the financial contracting models that we reviewed in the Introduction, is that the owner-manager does not raise $K_1 + K_2$ ex ante. This can be endogenized with the existence of an unlimited supply of fraudulent entrepreneurs who realize zero cash flow regardless of how much capital is sunk. Precisely, suppose that at $t = 1$ there is a publicly observable, but unverifiable signal whether the entrepreneur is such a "fly-by-night operator" (Rajan, 1992). Then, if the owner-manager had the unconditional right to decide on K_2, a fraudulent owner-manager could "blackmail" outside investors by demanding a sufficiently large transfer in return for paying *back* K_2. Conferring the right to stop refinancing to the investor, which is equivalent to the stipulated staging of financing, renders entry for such fraudulent entrepreneurs unprofitable. On the other hand, presently this feature of contracting exposes the firm to a hold-up problem. If *both* securities (R^1, R^2) were specified ex-ante, clearly the option to withhold refinancing, together with the distribution of bargaining power, would still ensure that R^2 is renegotiated in the investor's interest alone (i.e., as given in Proposition 1). A renegotiation-proof contract (R^1, R^2) could be interpreted also as a (single) convertible security, which exchanges senior financing R^1 for junior financing R^2 when additional capital, K_2, is injected.

As noted in the Introduction, the present case where the firm is locked-in may capture various forms of relationship financing. We relate our results to the respective empirical evidence in Section 5. Formally, such a "lock-in" can be obtained when there is an information asymmetry between the original investor and new investors. In fact, our model could be readily extended by introducing an additional layer of information asymmetry between the owner-manager and the original investor, on one side, and new investors in $t = 2$, on the other side. New investors would then face a "lemons problem" when being asked to fund all or a part of K_2. This may make access to new investors very costly or even impossible for the firm at $t = 2$ (cf. Sharpe, 1990; Rajan, 1992).[12]

4 The Case with a Weak Investor

We now consider the case in which the owner-manager has all the bargaining power also at the refinancing stage. We capture this, in analogy to the previous Section, by stipulating that she makes a take-it-or-leave-it offer R^2. This gives rise to a game of signaling, as at $t = 2$ the owner-manager is privately informed about the probability of the good state, q. Recall that we motivated this with the firm's need to raise fresh financing relatively quickly, so that an information asymmetry between insiders and outside investors can not be resolved in time. Again, we solve first for the equilibrium in the interim period, before turning to the optimal contract to raise initial financing at $t = 1$.

4.1 Refinancing (Weak Investor)

Game of Signaling A candidate for an equilibrium of the signaling game where each "type" q plays a pure strategy is a triple of functions $(R^2(q), \mu^*, \pi)$: $R^2(q)$ is the security issued by the owner-manager of type q, where we allow for $R^2(q) = \varnothing$ to capture the case where no new security is offered; μ^* is the investor's posterior belief, which maps the proposed security contract into the set of probability distributions over the type set $q \in [0, 1]$; and π represents the investor's decision to refinance the project, where $\pi : R^2 \rightarrow [0, 1]$ (with $\pi = 1$ corresponding to $d = Y$ and $\pi = 0$ corresponding to $d = N$). Our equilibrium concept is that of a Perfect Bayesian Equilibrium.

Efficient Refinancing Recall that the initial security, R^1, generates type-dependent outside options at the refinancing stage, both for the outside investor and for the owner-manager. This makes our signaling game different from previous security design papers such as Nachman and Noe (1994). In what follows, we show that this difference drastically changes the analysis. In particular, the efficient outcome may be obtained, and even though the outcome is pooling, there may not be cross-subsidization of less profitable types q.

[12] To incorporate this into our model, we suppose that the stipulated cash flow realizations only apply for some "type" $\phi = h$, while with positive probability the firm may be of some "type" $\phi = l$ with substantially worse cash flow realizations. The type ϕ is observed before the refinancing decision, but only by "insiders" (cf. also Inderst, Münnich, and Mueller (2007) for a formalization along these lines).

Proceeding in analogy to the case with a strong investor, we first define a security $R^2 = \widehat{R}^M$ that will allow efficient refinancing. (Here, the superscript M denotes the present case where the (owner-)manager makes the offer.) The security $\widehat{R}^M$ is defined by the requirement

$$v_Y(\widehat{R}^M, q) = v_N\left(R^1, q\right) \text{ for all } q.$$

Hence, with $\widehat{R}^M$ the investor would be indifferent between refinancing the firm and not refinancing it *for all* q. More explicitly, this obtains

$$\Delta\widehat{R}^M = \frac{(p_{NG} - p_{NB})}{(p_{YG} - p_{YB})}\Delta R^1, \tag{15}$$

$$\widehat{R}_l^M = R_l^1 + p_{NB}\Delta R^1 - p_{YB}\Delta\widehat{R}^M + K_2. \tag{16}$$

If such a security was accepted by the investor, it would allow the owner-manager to extract all of the surplus obtained from refinancing. The key feature of security $\widehat{R}^M$ is that it is "beliefs-free" for the investor: Regardless of the firm's profitability type, the investor is indifferent between retaining his old claim R^1 without refinancing and exchanging it for $\widehat{R}^M$ after investing additionally K_2. Clearly, if no such initial financing, R^1, were in place, as in standard models of security design under private information, this would be feasible only if the new security were riskless debt. Then - by definition - the value of the security would not depend on the firm's profitability. Instead, the value of $\widehat{R}^M$ does depend on the firm's type and thus on the owner-manager's private information. However, when there is initial financing in place, what matters is that the *difference* between the firm value under the new and the old security does not depend on the type.

We now argue that provided that $\widehat{R}^M$ is feasible, for given R^1, refinancing is obtained if and only if it is efficient, i.e., for all $q > q_{FB}$ but not for all $q < q_{FB}$, and that it is indeed obtained through issuing $R^2 = \widehat{R}^M$. As a first step in the argument, note that the investor will accept $\widehat{R}^M$ whenever it is offered. To be precise, observe that by marginally increasing either $\widehat{R}_l^M$ or $\Delta\widehat{R}^M$, the investor's preference for accepting can be made strict. This implies that any type $q > q_{FB}$ can ensure himself (arbitrarily close to) the full surplus from refinancing, so that there can be no "cross-subsidization" among types. This also implies that there is no refinancing for all $q < q_{FB}$, as the contract rules out cross-subsidization across types. Next, note that for given q there clearly exists more then one refinancing contract R^2 satisfying $v_Y(R^2, q) = v_N\left(R^1, q\right)$, so that for *this* type the investor is made indifferent between refinancing and not refinancing. However, if such a contract

R^2 were *not* equal to $\widehat{R}^M$, then by construction of $\widehat{R}^M$ there would be types $q' > q$ or types $q < q'$ for which the aforementioned cross-subsidization would now be feasible. This is why in equilibrium refinancing must be obtained with $R^2 = \widehat{R}^M$.[13]

Proposition 4 *Security $\widehat{R}^M$ is feasible at the refinancing stage $t = 2$ if*

$$x_l \geq R_l^1 + \frac{p_{YG}p_{NB} - p_{NG}p_{YB}}{p_{YG} - p_{YB}} \Delta R^1 + K_2. \tag{17}$$

Then, in the case with a weak investor, refinancing is obtained when $q > q_{FB}$ and not obtained when $q < q_{FB}$. Types $q_{FB} < q < 1$ uniquely offer $\widehat{R}^M$, where $\widehat{R}^M$ is flatter from the investor's perspective than the initial security R^1: $\widehat{R}_l^M \geq R_l^1$ and $\Delta\widehat{R}^M \leq \Delta R^1$ (the inequalities are strict if $\Delta R^1 > 0$).

Proof. See Appendix.

The existence of a "first-best" security $\widehat{R}^M$, provided that condition (17) is satisfied, distinguishes the outcome of our game of signaling from the previous security design literature where financing is raised under private information. This follows from the type-dependence of the outside option of the *investor*. In a standard model where no initial security is in place, the investor strictly prefers to finance a higher type, so that he is willing to accept a less favorable security. This makes it strictly preferable for *all* types to mimic a higher type. This is no longer the case when an initial security is in place. Then, also the value of the initial security strictly increases in the firm's profitability, making the investor less inclined to accept a new security in exchange for refinancing. Under the first-best security $\widehat{R}^M$ these two countervailing forces just balance.

Observe next that $\widehat{R}^M$ must be flatter than R^1. Refinancing increases the upside probability in the good state relative to that in the bad state. To ensure that the owner-manager can extract all of the net surplus for all q, the new contract must give the investor less from the upside and more when the low cash flow is realized.

Equilibrium Security When Refinancing is Inefficient If condition (17) is not satisfied, then $\widehat{R}^M$ is not feasible. Extracting the full surplus from refinancing and ensuring

[13] Strictly speaking, this argument does not apply at the boundaries $q = q_{FB}$ and $q = 1$, where incentive compatibility can be ensured with a flatter or steeper contract, respectively, provided that condition (17) is slack so that the construction is feasible. The realizations of $q = 0$ and $q = 1$ are, however, zero-probability events.

refinancing only when this is efficient is then no longer feasible for the owner-manager. In fact, there is no longer an equilibrium in which a type q offers some security R^2 and extracts the full surplus, while being the highest type to issue R^2.: There will be cross-subsidization in case refinancing is obtained.

To see this, suppose (17) does not hold. Consider some type $q = q^H$ and suppose that $v_Y(R^2, q^H) = v_N(R^1, q^H)$ holds for some security R^2. Now, it is no longer possible to make R^2 "flat enough" from the investor's perspective, implying that $v_Y(R^2, q)$ intersects with $v_N(R^1, q)$ from below at q^H. Therefore, for all $q < q^H$ that offer this contract, the investor would realize strictly less from providing refinancing than under his outside option of not doing so. Clearly, any such $q < q^H$ would then strictly prefer R^2 over any other contract that would allow this type to extract just the respective surplus from refinancing. Hence, if q^H is the highest type (in a pool) issuing R^2, he will extract strictly less than "his surplus". A key insight that is shared with much of the literature on security design with adverse selection (see, in particular, Nachman and Noe, 1994) is that the degree of such "cross-subsidization" is lowest when, in a given pool, the respective security is debt. Intuitively, the difference $v_Y(R^2, q) - v_N(R^1, q)$, which is strictly increasing in q, becomes flatter if debt is issued, as debt is least "information-sensitive" to the private information q.

Following this literature (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999; or DeMarzo et al., 2005), we now apply criterion D1 to refine the out-of-equilibrium beliefs (Cho and Kreps, 1987; Ramey, 1996). Roughly speaking, if type q' has a weak incentive to deviate to some security $\widetilde{R}^2$, while type q has a strict incentive, then the investor should put probability zero on type q' making this deviation. (A formal definition is contained in the proof of the subsequent proposition.) Then, in equilibrium all type must offer a (pooling) debt contract as, otherwise, the highest type in a pool would credibly deviate by offering a flatter refinancing contract to the investor.

Proposition 5 *Take the case with a weak investor at $t = 2$, where we apply the refinement D1 for the resulting game of signaling. Suppose condition (17) does not hold, so that $\widehat{R}^M$ is not feasible and there will be cross-subsidization in case of refinancing. Then, in any refined equilibrium there is a cutoff q_M^* so that all types $q < q_M^*$ do not refinance while all types $q > q_M^*$ refinance. Refinancing is obtained for all $q \geq q_M^*$ through the same debt contract R^2 (i.e., $R_l^2 = x_l$).*

Proof. See Appendix.

Note that Proposition 4 and 5 joint imply that the refinancing security is now always flatter than the original security R^1, irrespective of whether there is cross-subsidization or not. Define next for given initial security R^1 a pooling debt security $R^2 = R^P$ for which the investor is just indifferent to refinancing: $R^P = \left(x_l, \Delta R^P\right)$ and q_M^* jointly satisfy

$$\begin{aligned}\int_{q_M^*}^1 \left[v_Y\left(R^P, q\right) - v_N\left(R^1, q\right)\right] \frac{dF(q)}{1 - F(q_M^*)} &= 0, \\ u_Y\left(R^P, q_M^*\right) - u_N\left(R^1, q_M^*\right) &= 0.\end{aligned} \tag{18}$$

Note that $q_M^* < q_{FB}$, as there is cross-subsidization of lower types under refinancing. Hence, there is overinvestment at the refinancing stage. The outcome where all $q \geq q_M^*$ pool at this particular (break-even) debt contract can be supported by beliefs that satisfy the imposed refinement D1. However, D1 does not eliminate other pooling equilibria with debt where the investor is left with a strictly positive "rent" under refinancing: D1 uniquely pins down the *shape* of the refinancing security, but not the level. In what follows, we now impose the common restriction that the investor just breaks even, so that the equilibrium at $t = 2$ is uniquely pinned down by (18), provided that condition (17) does not hold.[14]

4.2 Raising Initial Finance (Weak Investor)

Recall that at $t = 1$ there is no private information and financing can be raised at competitive terms. The owner-manager thus maximizes

$$\int_0^{q_M^*} \left(s_N(q) - v_N\left(R^1, q\right)\right) dF(q) + \int_{q_M^*}^1 \left(s_Y(q) - v_Y\left(R^2, q\right)\right) dF(q),$$

subject to the ex ante participation constraints of the investor

$$\int_0^{q_M^*} v_N\left(R^1, q\right) dF(q) + \int_{q_M^*}^1 v_Y\left(R^2, q\right) dF(q) \geq K_1, \tag{19}$$

where q_M^* and $R^2 = R^P$ are determined either from (18), when (17) does not hold, or from $q_M^* = q_{FB}$ and $R^2 = \widehat{R}^M$, when (17) holds. Suppose first that there is inefficiency at the

[14] In fact, it can be shown that this is also the unique equilibrium outcome if there is competition by outside investors at $t = 2$. Precisely, suppose there are at least two new outside investors who could offer to refinance K_2 and, at the same time, to buy out the incumbent investor. To preserve the bargaining power of the owner-manager, assume that the owner-manager must agree first to such a proposal, before it is passed on to the incumbent investor. Then, R^P is the unique outcome in this game of competition.

refinancing stage. We present below a simple condition when this is the case in equilibrium and show that then (19) binds. The owner-manager is the residual claimant in $t=1$. His aim is thus to maximize the expected surplus at $t=1$. That is, the security she offers in the initial period should be designed so that the gap between the cutoffs q_M^* and q_{FB} is minimal.

Proposition 6 *Suppose there is overinvestment at the refinancing stage. Then, it is uniquely optimal for the firm to raise initial financing through a levered equity contract,* $R_l^1=0$.

Proof. See Appendix.

Levered equity is the steepest security from the investor's perspective and thus the flattest security from the owner-manager's perspective. Recall that, presently, with a weak investor the inefficiency at $t=2$ is caused by overinvestment, as also types $[q_M^*, q_{FB})$ receive refinancing under the pooling (debt) contract. Raising the initial amount K_1 through levered equity financing preserves the firm's "debt capacity". That is, the refinancing security R^2 can then be made flatter *relative* to the initial security R^1. This limits cross-subsidization and thus improves efficiency by pushing up the marginal type q_M^*.

Recall now that when the first-best investment is feasible at $t=2$, so that the cutoff satisfies $q_M^*=q_{FB}$, the "first-best security" $R^2=\widehat{R}^M$ is flatter than R^1. Proposition 7 derives a condition when efficiency is feasible. We have thus arrived at the following result.

Proposition 7 *Take the case with a weak investor at* $t=2$. *Then, the first-best refinancing outcome* ($q_M^*=q_{FB}$) *is obtained when*

$$x_l \geq K_2+\frac{p_{NB}p_{YG}-p_{YB}p_{NG}}{(p_{YG}-p_{YB})\,p_N(\widehat{q})}K_1+\frac{(p_{NG}-p_{NB})\,p_Y(\widehat{q})}{(p_{YG}-p_{YB})\,p_N(\widehat{q})}\max\left(0,K_1-p_N(\widehat{q})\Delta x\right), \quad (20)$$

while otherwise there is overinvestment with $q_M^*<q_{FB}$. *In either case, the security that is held by outside investors after refinancing at* $t=2$ *is unambiguously flatter from the investor's perspective than the initial security* ($\Delta R^2<\Delta R^1$ *and* $R_l^2>R_l^1$).

Proof. See Appendix.

The intuition for condition (20) is straightforward given the previous discussion and condition (17), which showed when $\widehat{R}^M$ is feasible for given R^1. If x_l is large enough, the owner-manager can ensure that the investor just breaks even while at the same time making R^1 sufficiently steep, so that subsequently the "first-best" security $\widehat{R}^M$ is feasible at $t = 2$. When condition (20) holds strictly, there is again some leeway to choose the initial security R^1, while still ensuring that subsequent financing is raised efficiently. Still, for any optimal choice R^1, refinancing will lead to a strictly flatter security for the investor: He will receive less from the upside, but he is more protected on the downside.

5 Implications and Evidence

We argued in Section 3 that the strong-investor case may better describe situations in which the firm enters into a relationship with an investor. We also discussed how the characterized securities (R^1, R^2) could then describe a sequence of renegotiation-proof contracts. In Section 4, we considered the polar case in which it is the owner-manager who has full bargaining power at the refinancing stage. One interpretation that we offered so far is that this represents a competitive market for financing also at $t = 2$, as the incumbent investor has no privileged information. Alternatively, the owner-manager's bargaining power at $t = 2$ could come from her threat to withhold essential human capital that is needed to grow the business (cf. Hart and Moore, 1994). Hence, the weak-investor case may also describe the financing of firms that depend heavily on human capital provided by insiders. Outside investors may have low bargaining power also in countries with weak protection of their rights or with weak legal enforcement of these rights. We explore the respective interpretations in more detail in what follows, deriving various implications from our model of financial contracting.

Our stylized model of a firm's dynamic contracting problem obtains implications both for a firm's choice of financing under asymmetric information and for its change of leverage over time. The respective results in Propositions 3 and 7 imply that the change in leverage depends crucially on whether the firm has access to competitive financing also when it must raise financing under asymmetric information (at short notice) or whether it is then locked into a relationship with an incumbent outside investor.

5.1 Start-Up Financing and Young Firm Financing

A suitable environment to test the contrasting predictions is the financing of young firms with growth potential. In this case, as noted above, we can take the two-stage nature of our model literally. Initially, when marketing her idea, we stipulate that the owner-manager can bridge any information asymmetry vis-à-vis outside investors.[15] At a later stage, however, when fresh financing has to be raised, probably at short notice, the information gap with outside investors may have widened as only the owner-manager is involved in the firm's day-to-day operations. Also, there may then be insufficient time to credibly divulge all relevant information to investors. We now argue that the contrasting implications of our model are particularly suitable to match the rich evidence from the recent literature on start-up and young firm financing.

Following the empirical contributions of Gompers (1995) and Kaplan and Strömberg (2003), several theoretical papers have tried to explain why venture capitalists provide funds in exchange for senior securities with the option of converting them into junior ones (e.g., Berglöf, 1994; Cornelli and Yosha, 1997; Hellmann, 2006). Our results for the *strong-investor case* are consistent with this observation (cf. our previous interpretations in terms of convertible securities), albeit in contrast to the extant theoretical literature our characterization focuses on the distribution of cash flows rather than the contingent allocation of control rights. Our model also has richer implications as these convertible securities do not always arise in equilibrium. The resulting contrasting implications allow us to organize the richer findings in the recent empirical literature. This literature has taken a more differentiated view, showing that convertible debt and preferred equity are less common in start-up finance under different legal environments (Lerner and Schoar, 2005; Cumming 2008; Kaplan et al., 2007). Lerner and Schoar (2005) have shown that while private equity firms in countries with common law or strong legal enforcement often use convertible preferred stock, common stock is the favored instrument in weak enforcement and civil law countries. This is consistent with our results, as we predict a switch from senior to junior financing when the investor is strong in $t = 2$ and the opposite when the investor is weak. Also Kaplan et al. (2007) find that in countries with weaker creditor

[15] In fact, it has sometimes been argued that at this stage it is often the investor who, due to his industry knowledge as a "long-run" player, may be better able to gauge the prospects of a business plan (e.g., Inderst and Müller 2006).

rights equity is more frequently issued in first rounds. Further, the more successful (and thus possibly stronger) firms that have issued equity in earlier rounds switch to more senior securities in later rounds.[16]

Hellmann et al. (2008) analyze banks' private equity investments in start-up firms. They find that banks direct their *equity* investments towards later stages of the development of start-ups. The same banks are then significantly more likely to subsequently grant a loan (i.e. debt financing) to these firms. This finding, which is confirmed by Fang et al. (2010), is in line with our predictions when we apply the following reasoning. In contrast to specialized venture capitalists, banks may lack specific industry knowledge as well as the management skills that are required, in particular, at the early stage. One may thus presume that firms obtaining risk capital from banks may be less likely to find themselves locked-in in the future. This translates to our weak-investor case, for which the prediction of a shift from equity to debt financing is in line with the aforementioned empirical observations.

5.2 Long-Term vs. Short-Term Financial Strategy

As noted in the Introduction, though our stylized model has only two periods, we interpret our results also in terms of a firm's long-term and short-term financial strategy. Our motivation for this is as follows. We stipulate that information asymmetry should be more relevant when financing is needed at short notice, e.g., to realize an investment opportunity that is open only for a short time. Instead, when a firm chooses its long-term (or target) financial structure, this choice should be plagued less by information asymmetry. The firm should then have time to credibly bridge or at least significantly narrow an informational gap vis-à-vis outside investors. These two choices correspond to the security design problems in $t = 2$ and $t = 1$ of our model. In particular, the security that is held by outside investors after refinancing in $t = 2$ may represent a firm's temporary deviation from its long-term financial strategy.

When we take this perspective, an application of our weak-investor case yields the following implications. Large firms with access to capital markets may choose "armslength" financing as a long-term capital strategy, so that even in $t = 2$ of our model they will face a competitive financial market (albeit one plagued by information asymmetry). Fama

[16] Kaplan et al. (2007) interpret this as a sign of learning that US-style contracts are more efficient.

and French (2005) find that equity issuance is often observed in companies that are not under duress. They interpret this as a violation of Myers and Majluf's (1984) pecking order theory (cf. also Leary and Roberts, 2010). Our model shows that this allows firms to preserve their "debt capacity" (cf. Lemmon and Zender, 2009), and we predict that, in line with the pecking order, debt financing will be chosen if financing is needed at short notice. Put differently, firms that expect to have continuing access to a competitive financial market should use equity issuances to bring their financing in line with their target capital structure, while short-term deviations are observed jointly with fresh debt financing.

We obtain strikingly different predictions for firms that raise financing from (relationship) investors who hold bargaining power in the future, when additional financing is needed at relatively short notice. These firms should choose high leverage as their target capital structure, which preserves "equity capacity". Deviations from their long-term capital structure should thus involve the issuance of equity rather than debt (or, likewise, the conversion of outstanding securities into more junior claims). Interestingly, for these firms debt would thus represent the preferred source of financing when information asymmetries are not an issue, which is in line with the survey evidence in Graham and Harvey (2001). It is further the predominant choice for small firms that often obtain loans from a single bank with which they have close ties (Petersen and Rajan, 1994; Detragiache et al. 2000). The findings of Leary and Roberts (2010) and Gomes and Phillips (2005) suggest that smaller growth firms will indeed issue debt when asymmetric information is not a factor, so as to preserve "equity capacity" for the long run.

6 Conclusion

We develop a dynamic theory of a firm's security design and optimal capital structure. The key linkage between the firm's choice of initial financing, which is raised "for the long-term" under symmetric information, and its subsequent financing under asymmetric information is that the former affects the "outside options" for both insiders and outside investors when new financing must be raised. We find that the model's implications for the optimal financial structure and it's change over time differ sharply depending on whether the bargaining power at the refinancing stage lies (more) with the firm, as it faces a competitive capital market, or with initial investors.

If incumbent investors have bargaining power at the refinancing stage, inefficiency in the form of underinvestment may arise, as they attempt to extract higher "rents" from better informed insiders (the "owner-manager" in our model). Instead, a problem of overinvestment is likely if bargaining power lies with the better informed insiders.

If initial investors have bargaining power when financing is needed in the future, the firm will raise additional financing through reducing leverage. Intuitively, in this case a firm's long-term (target) capital structure preserves "equity capacity", as this reduces underinvestment in the future. In contrast, if bargaining power is in the hands of the firm also when future financing is needed, then the initial (or long-term) leverage decision serves to reduce the subsequent overinvestment problem: The firm preserves its "debt capacity", as it seeks refinancing through issuing debt in the future.

We show that our richer implications, in contrast to most standard theories of security design under asymmetric information, are largely in line with the sometimes contrasting recent evidence in the literature. Our polar cases with strong or weak investors may shed light on cross-country differences in start-up and young firm financing, as well as differences between early- and later-stage financing. We also related our results to a firm's choice of target financial structure vis-à-vis temporary deviations that are due to its need to raise financing at short term under problems of asymmetric information. Admittedly, these implications are somewhat tentative as our model is highly stylized, allowing only for two periods. Hence, this perspective could be explored further in a model with an open time horizon. Further, though we showed how our restriction on the amount of financing that is raised initially (or, for that matter, for the long term) can be endogenized, a firm may hold free cash as part of its optimal financial strategy when the agency problems that this engenders are not too large.

References

Aghion, Philippe, and Patrick Bolton, 1992, An incomplete contracts approach to financial contracting, *Review of Economic Studies* 59, 473-494.

Axelson, Ulf, Per Strömberg, and Michael S. Weisbach, 2009, Why are buyouts levered? The financial structure of private equity funds, *Journal of Finance* 64, 1549–1582.

Berglöf, Eric, 1994, A control theory of venture capital finance, *Journal of Law, Economics, and Organization* 10, 247–267.

Bias, Bruno, and Thomas Mariotti, 2005, Strategic liquidity supply and security design, *Review of Economic Studies* 72, 615–649.

Burkart, Mike, and Samuel Lee, 2011, Smart buyers, Working paper Stockholm School of Economics and NYU.

Cho, In-Koo, and David M. Kreps, 1987, Signaling games and stable equilibria, *Quarterly Journal of Economics* 102, 179–221.

Cornelli, Francesca, and Oved Yosha, 2003, Stage financing and the role of convertible securities, *Review of Economic Studies* 70, 1–32.

Cumming, Douglas, 2005, Capital structure in venture finance, *Journal of Corporate Finance* 11, 550–585.

Cumming, Douglas, 2008, Contracts and exits in venture capital finance, *Review of Financial Studies* 21, 1948–1982.

Degryse, Hans, and Steven Ongena, 2008, Competition and regulation in the banking sector: A review of the empirical evidence on the sources of bank rents, in Anjan V. Thakor, and Arnoud W. A. Boot, eds. *Handbook of Financial Intermediation and Banking* (Elsevier, Amsterdam).

DeMarzo, Peter M., and Darrell Duffie, 1999, A liquidity-based model of security design, *Econometrica* 67, 65–99.

DeMarzo, Peter M., Ilan Kremer, and Andrzej Skrzypacz, 2005, Bidding with securities: Auctions and security design, *American Economic Review* 95, 936–959.

Detragiache, Enrica, Paolo Garella, and Luigi Guiso, 2000, Multiple versus single banking relationships: Theory and evidence, *Journal of Finance* 55, 1133–1161.

Fama, Eugene F., and Kenneth R. French, 2005. Financing decisions: Who issues stock?, *Journal of Financial Economics* 76, 549–582.

Fang, Lily H., Victoria Ivashina, Josh Lerner, 2010, Unstable Equity? Combining banking with private equity investing, SSRN Working paper INSEAD and Harvard University.

Frank, Murray Z., and Vidhan K. Goyal, 2003, Testing the pecking order theory of capital structure, *Journal of Financial Economics* 67, 217–248.

Gomes, Armando, and Gordon Phillips, 2005, Why do public firms issue private and public securities?, Working paper Washington University-St. Louis and University of Maryland.

Gompers, Paul, 1995, Optimal investment, monitoring, and the staging of venture capital, *Journal of Finance* 50, 1461–1489.

Graham, John R., and Campbell Harvey, 2001, The theory and practice of corporate finance: Evidence from the field, *Journal of Financial Economics* 60, 187-243.

Hart, Oliver D., and John Moore, 1994, A theory of debt based on the inalienability of human capital, *Quarterly Journal of Economics* 109, 841–879.

Hellmann, Thomas, 2006, IPOs, acquisitions, and the use of convertible securities in venture capital, *Journal of Financial Economics* 81, 649–679.

Hellmann, Thomas, Laura Lindsey, and Manju Puri, 2008, Building relationships early: Banks in venture capital, *Review of Financial Studies* 21, 513–541.

Hennessy, Christopher A., and Toni M. Whited, 2005, Debt dynamics, *Journal of Finance* 60, 1129–1165.

Inderst, Roman, and Holger Müller, 2006, Informed lending and security design, *Journal of Finance* 61, 2137–2162.

Inderst, Roman, Holger Müller, and Felix Münnich 2007, Financing a portfolio of projects, *Review of Financial Studies* 20, 1289–1325.

Kaplan, Steven N., and Per Strömberg, 2003, Financial contracting theory meets the real world: An empirical analysis of venture capital contracts, *Review of Economic Studies* 70, 281–315.

Kaplan, Steven N., Frederic Martel, and Per Strömberg, 2007, How do legal differences and experience affect financial contracts?, *Journal of Financial Intermediation* 16, 273–311.

Leary, Mark T., and Michael R. Roberts, 2010, The pecking order, debt capacity, and information asymmetry, *Journal of Financial Economics* 95, 332-355.

Lemmon, Michael L., and Jaime F. Zender, 2009, Debt capacity and tests of capital structure, *Journal of Financial and Quantitative Analysis*, forthcoming.

Lerner, Josh, and Antoinette Schoar, 2005, Does legal enforcement affect financial transactions? The contractual channel in private equity, *Quarterly Journal of Economics* 120, 223–246.

Nachman, David C., and Thomas H. Noe, 1994, Optimal design of securities under asymmetric information, *Review of Financial Studies* 7, 1–44.

Miao, Jianjun, 2005, Optimal capital structure and industry dynamics, *Journal of Finance* 60, 2621-2659.

Myers, Stewart, and Nicholas Majluf, 1984. Corporate financing and investment decisions when firms have information that investors do not have, *Journal of Financial Economics* 13, 187–221.

Petersen, Mitchell A., and Raghuram G. Rajan, 1994, The benefits of lending relationships: Evidence from small business data, *Journal of Finance* 49, 3–37.

Rajan, Raghuram G., 1992, Insiders and outsiders: The choice between informed and arm's-length debt, *Journal of Finance* 47, 1367–1400.

Ramey, Garey, 1996, D1 signaling equilibria with multiple signals and a continuum of types, *Journal of Economic Theory* 69, 508–531.

Sharpe, Steven A., 1990, Asymmetric information, bank lending and implicit contracts: A stylized model of customer relationships, *Journal of Finance* 45, 1069–1087.

Shyam-Sunder, Lakshmi, and Stewert C. Myers, 1999, Testing static trade-off against pecking order models of capital structure, *Journal of Financial Economics* 51, 219–244.

Tirole, Jean, 2011, Overcoming adverse selection: How public intervention can restore market functioning, *American Economic Review*, forthcoming.

Appendix A Omitted Proofs

Proof of Proposition 1. The proof follows from a sequence of auxiliary results.

Claim 1. *The first-best security $\widehat{R}$ is feasible if and only if condition (9) holds.*

Proof. Note first that if the initial security R^1 is feasible, then from $\Delta x - \Delta R^1 \geq 0$ and from the construction of $\Delta\widehat{R}$ in (5) we also have that $\Delta x - \Delta\widehat{R} \geq 0$. Further, as condition (1) implies that $p_{YG} - p_{YB} > p_{NG} - p_{NB}$, we have from (5) that $\Delta\widehat{R} \geq 0$. To see next that $\widehat{R}_l \leq x_l$ holds, we substitute (5) into (6) and obtain

$$\widehat{R}_l = R_l^1 - \left(\frac{p_{YG}p_{NB} - p_{YB}p_{NG}}{p_{YG} - p_{YB}}\right)\left(\Delta x - \Delta R^1\right). \tag{A.1}$$

This implies from (1) that $\widehat{R}_l < R_l^1$ and thus also $\widehat{R}_l < x_l$, given that R^1 was feasible. The remaining condition is thus that $\widehat{R}_l \geq 0$, which from (A.1) is just condition (9). From this it also follows that (9) is necessary for $\widehat{R}$ to be feasible. **Q.E.D.**

The next claim establishes that by optimality of R^2, the set of owner-manager types that accepts, $q \in A$, is always characterized by a cutoff q^*. We argue to a contradiction, showing that if there existed a security R^2 so that the owner-manager would prefer acceptance for low but *not* for high q, then the first-best contract $\widehat{R}$ would be feasible, instead. Then, as argued in the main text, it is clearly optimal to offer $\widehat{R}$.

Claim 2. *If a security R^2 satisfying $u_Y(R^2, 0) > u_N(R^1, 0)$ together with $u_Y(R^2, 1) < u_N(R^1, 1)$ is feasible, then also the first-best security $\widehat{R}$ is feasible.*

Proof. Note first that from the assumed inequalities $u_Y(R^2, 0) > u_N(R^1, 0)$ (owner-manager prefers refinancing for $q = 0$) and $u_N(R^1, 1) > u_Y(\widehat{R}^2, 1)$ (owner-manager prefers no-refinancing for $q = 1$), $\Delta\widehat{R} < \Delta R^2$ must hold to ensure that the slope of $u_Y(R^2, q)$ is strictly smaller than that of $u_Y(\widehat{R}, q)$. But then $u_Y(R^2, 0) > u_Y(\widehat{R}, 0)$ implies that $R_l^2 < \widehat{R}_l$. By the assumed feasibility of R^2, we have from this that $\widehat{R}_l > 0$, so that (9) holds strictly. **Q.E.D.**

From Claims 1-2 refinancing takes place whenever $q \geq q^*$ (with $q^* = q_{FB}$ if $\widehat{R}$ is feasible). It is straightforward to rule out optimality of the case $q^* = 1$ (zero probability of refinancing). If $q^* < 1$, then the cutoff is pinned down by the requirement that $u_Y(R^2, q^*) = u_N(R^1, q^*)$ (cf. also (7)).

Claim 3. *Levered-equity with $R_l^2 = 0$ is the uniquely optimal security for the investor if the first-best security $\widehat{R}$ is not feasible.*

Proof. We argue to a contradiction. Suppose that, so as to implement some $q^* \in [0,1]$, another security R^2 with $R_l^2 > 0$ were optimal. Choose now $\widetilde{R}^2 = (0, \Delta\widetilde{R}^2)$ so that $u_Y(\widetilde{R}^2, q^*) = u_N(R^1, q^*)$, which implies that the owner-manager's acceptance set, $[q^*, 1]$, remains unchanged, while at q^* the investor's *conditional* expected payoff does not change: $v_Y(\widetilde{R}^2, q^*) = v_Y(R^2, q^*)$. However, as $u_Y(\widetilde{R}^2, q^*) = u_Y(R^2, q^*)$ together with $\widetilde{R}_l^2 = 0 < R_l^2$ must imply that $\Delta\widetilde{R}^2 > \Delta R^2$, we have that $v_Y(\widetilde{R}^2, q) - v_Y(R^2, q) > 0$ holds for all $q > q^*$. Thus, provided it is feasible, the investor is indeed strictly better off under the newly constructed contract $\widetilde{R}^2$.

It remains to show that $\widetilde{R}^2$ is indeed feasible. By the assumed feasibility of R^2 and construction of $\widetilde{R}^2$, this is the case if $\Delta\widetilde{R}^2 \leq \Delta x$. (The other feasibility restrictions on $\widetilde{R}^2$ are satisfied by feasibility of R^2.) From $u_Y(\widetilde{R}^2, q^*) = u_Y(R^2, q^*)$ and $\widetilde{R}_l^2 = 0$, we can obtain

$$\Delta\widetilde{R}^2 = \frac{R_l^2}{p_{YB} + q^*(p_{YG} - p_{YB})} + \Delta R^2,$$

so that $\Delta\widetilde{R}^2 \leq \Delta x$ holds whenever

$$0 \leq -R_l^2 + (p_{YB} + q^*(p_{YG} - p_{YB}))\left(\Delta x - \Delta R^2\right). \tag{A.2}$$

However, (A.2) is implied by the assumption that the first-best security is not feasible, i.e., that (9) does not hold. To see this, note first that from the definition of q^*, i.e. $u_Y(R^2, q^*) = u_N(R^1, q^*)$, condition (A.2) is equivalent to

$$0 \leq -R_l^1 + (p_{NB} + q^*(p_{NG} - p_{NB}))\left(\Delta x - \Delta R^1\right). \tag{A.3}$$

As, by assumption, $\widehat{R}$ is not feasible, it holds from transforming the "first-best condition" (9) that

$$\begin{aligned} 0 &< -R_l^1 + \left(\frac{p_{YG}p_{NB} - p_{YB}p_{NG}}{p_{YG} - p_{YB}}\right)\left(\Delta x - \Delta R^1\right) \\ &< -R_l^1 + (p_{NB} + q^*(p_{NG} - p_{NB}))\left(\Delta x - \Delta R^1\right), \end{aligned}$$

where the last inequality holds for *any* q^*. But this is just what we needed to show (condition (A.3)). **Q.E.D.**

To conclude the proof of Proposition 1, we solve the investor's program when $\widehat{R}$ is not feasible. For this observe that from the indifference condition of the owner-manager at q^*, (7), we have that

$$\Delta R^2 = \Delta x - \frac{R_l^2 - R_l^1 + [p_{NB} + q^* (p_{NG} - p_{NB})] (\Delta x - \Delta R^1)}{p_{YB} + q^* (p_{YG} - p_{YB})}, \tag{A.4}$$

from which we obtain explicitly

$$\frac{d\Delta R^2}{dq^*} = \frac{(R_l^2 - R_l^1)(p_{YG} - p_{YB}) + (p_{NB}p_{YG} - p_{YB}p_{NG})(\Delta x - \Delta R^1)}{[p_{YB} + q^* (p_{YG} - p_{YB})]^2} > 0, \tag{A.5}$$

where the inequality follows as $R_l^2 = 0$ when (9) does not hold.

We can next substitute for the acceptance set $A = [q^*, 1]$ into the investor's objective function (3), where q^* is given by the indifference condition for the owner-manager (cf. condition (7)). Differentiating with respect to q^*, we have the first-order condition (cf. also (10))

$$-[s_Y(q^*) - s_N(q^*)] f(q^*) + \frac{d\Delta R^2}{dq^*} \int_{q^*}^{1} \frac{dv_Y(R^2, q)}{d\Delta R^2} dF(q) = 0,$$

where the first term follows from $s_d(q) = u_d(R^t, q) + v_d(R^t, q)$ and (7). As $\frac{d\Delta R^2}{dq^*} > 0$,

$$\frac{dv_Y(R^2, q)}{d\Delta R^2} = p_{YB} + q(p_{YG} - p_{YB}),$$

while $s_Y(q^*) - s_N(q^*)$ is strictly increasing and equal to zero when $q^* = q_{FB}$, we have that $q^* > q_{FB}$. **Q.E.D.**

Proof of Proposition 2. The proof is by contradiction. Suppose that R^1 with $R_l^1 < x_l$ were optimal and that there is inefficiency at $t = 2$. By Proposition 1 the investor chooses a security $R^2 = (0, \Delta R^2)$ that implements a cutoff $q^*_{old} > q_{FB}$. Note that we relegate to the end of the proof the argument why, in the equilibrium of the whole game, the investor must always choose the most efficient cutoff from his optimal correspondence and thus plays a pure strategy. We proceed in several steps.

Step 1. We start by constructing $\widetilde{R}^1 = (x_l, \Delta\widetilde{R}^1)$ together with $\widetilde{R}^2 = (0, \Delta\widetilde{R}^2)$ so that two conditions are satisfied: The owner-manager is still indifferent at his old cutoff q^*_{old}

and, holding this cutoff fixed, the ex ante payoff for both parties stays the same. By construction, it then holds that

$$0=\int_{0}^{q^*_{old}}\left[v_N(\widetilde{R}^1,q)-v_N(R^1,q)\right]dF\left(q\right)+\int_{q^*_{old}}^{1}\left[v_Y(\widetilde{R}^2,q)-v_Y(R^2,q)\right]dF\left(q\right), \quad \text{(A.6)}$$

together with $u_Y(R^2,q^*_{old})=u_N(R^1,q^*_{old})$ and $u_Y(\widetilde{R}^2,q^*_{old})=u_N(\widetilde{R}^1,q^*_{old})$. To ease exposition, let

$$\begin{aligned}\widehat{p}_N \quad &: \quad = p_{NB}+(p_{NG}-p_{NB})\int_0^{q^*_{old}} q\frac{dF\left(q\right)}{F\left(q^*_{old}\right)},\\ \widehat{p}_Y \quad &: \quad = p_{YB}+(p_{YG}-p_{YB})\int_{q^*_{old}}^1 q\frac{dF\left(q\right)}{1-F\left(q^*_{old}\right)}.\end{aligned}$$

Further, let $p_d\left(q\right):=p_{dB}+q\left(p_{NG}-p_{NB}\right)$ be defined as in (13) in the main text. Recall also that, for given q^* and R^1, ΔR^2 is given in (A.4). Plugging into (A.6) we have

$$\begin{aligned}0 \quad = \quad & \left(x_l-R_l^1+\widehat{p}_N\left(\Delta\widetilde{R}^1-\Delta R^1\right)\right)F\left(q^*_{old}\right)\\ &+\frac{\widehat{p}_Y}{p_Y(q^*_{old})}\left(x_l-R_l^1+p_N(q^*_{old})\left(\Delta\widetilde{R}^1-\Delta R^1\right)\right)\left(1-F\left(q^*_{old}\right)\right),\end{aligned}$$

from which we can express $\Delta\widetilde{R}^1$ as

$$\Delta\widetilde{R}^1=\Delta R^1-\left(\frac{x_l-R_l^1}{\widehat{p}_N}\right)\left(\frac{p_Y(q^*_{old})F\left(q^*_{old}\right)+\widehat{p}_Y\left(1-F\left(q^*_{old}\right)\right)}{p_Y(q^*_{old})F\left(q^*_{old}\right)+\frac{p_N(q^*_{old})}{\widehat{p}_N}\widehat{p}_Y\left(1-F\left(q^*_{old}\right)\right)}\right). \quad \text{(A.7)}$$

Step 2. We now show that, if offered $\widetilde{R}^1$ in the initial period, the investor will actually offer a different security $\overline{R}^2\neq\widetilde{R}^2$ at $t=2$ that implements a strictly lower cutoff. For this purpose we look at the expected payoff of the investor at $t=2$ when he is faced with R^1 or $\widetilde{R}^1$, respectively, and then apply monotone comparative statics.

As the second security is levered equity with $R_l^2=\widetilde{R}_l^2=0$, the indifference condition of the owner-manager at a cutoff q^* gives the respective value ΔR^2 as a unique function of R^1 and q^* only (cf. (A.4)). We use $\Delta R^2\left(q^*,R^1\right)$ and $\Delta R^2(q^*,\widetilde{R}^1)$, making thereby explicit that $\Delta R^2(\cdot)$ presently denotes a function. Next, we define the investor's expected payoff at $t=2$ for some q^* and an initial contract R^1 by

$$V\left(q^*,R^1\right):=\int_0^{q^*}v_N\left(R^1,q\right)dF\left(q\right)+\int_{q^*}^1 v_Y\left(R^2,q\right)dF\left(q\right). \quad \text{(A.8)}$$

Defining $V(q^*, \widetilde{R}^1)$ accordingly, we now show that the difference $V(q^*, \widetilde{R}^1) - V(q^*, R^1)$ is decreasing in q^*. (Importantly, note that q^* is *not* an optimal selection from the investor's optimization problem at this point.) After some transformations we have

$$\begin{aligned} & \frac{d}{dq^*}\left[V(q^*, \widetilde{R}^1) - V\left(q^*, R^1\right)\right] \qquad \text{(A.9)} \\ = & \int_{q^*}^{1} p_Y(q)\left(\frac{d\Delta R^2(q^*, \widetilde{R}^1)}{dq^*} - \frac{d\Delta R^2(q^*, R^1)}{dq^*}\right) dF(q). \end{aligned}$$

Next, using (A.5) and (A.7), we obtain an explicit expression for the second term under the integral in (A.9). Importantly, observe that $\widetilde{R}^1$ is defined as a function of q^*_{old} and *not* q^*. We have

$$\begin{aligned} & \frac{d\Delta R^2(q^*, \widetilde{R}^1)}{dq^*} - \frac{d\Delta R^2(q^*, R^1)}{dq^*} \\ = & -\frac{(x_l - R_l^1)(p_{YG} - p_{YB}) + (p_{NB}p_{YG} - p_{YB}p_{NG})\left(\Delta\widetilde{R}^1 - \Delta R^1\right)}{p_Y(q^*)^2} \\ = & \frac{-(x_l - R_l^1)(p_{YG} - p_{YB})}{p_Y(q^*)^2} \\ & \times\left(1 - \frac{(p_{NB}p_{YG} - p_{YB}p_{NG})}{(p_{YG} - p_{YB})\widehat{p}_N}\frac{p_Y(q^*_{old})F(q^*_{old}) + \widehat{p}_Y(1 - F(q^*_{old}))}{p_Y(q^*_{old})F(q^*_{old}) + \frac{p_N(q^*_{old})}{\widehat{p}_N}\widehat{p}_Y(1 - F(q^*_{old}))}\right) \\ < & \frac{-(x_l - R_l^1)(p_{YG} - p_{YB})}{p_Y(q^*)^2}\left(1 - \frac{(p_{NB}p_{YG} - p_{YB}p_{NG})}{(p_{YG} - p_{YB})\widehat{p}_N}\right) < 0, \end{aligned}$$

where for the first inequality we use that $p_N(q^*_{old})/\widehat{p}_N > 1$, and for the second inequality we use that $\widehat{p}_N > p_{NB}$. From (A.9), it follows, therefore, that

$$\frac{dV(q^*, \widetilde{R}^1)}{dq^*} < \frac{dV(q^*, R^1)}{dq^*}.$$

Thus, the difference $V(q^*, \widetilde{R}^1) - V(q^*, R^1)$ decreases in q^*. By standard monotone selection arguments, strictly decreasing differences imply the following: Any optimal cutoff q^*_{new} that the investor chooses given $\widetilde{R}^1$ is lower than any optimal cutoff q^*_{old} that he selects given R^1, so that $q^*_{new} < q^*_{old}$.

Step 3. In this step we show that the owner-manager is indeed better off with the considered deviation. Observe first that by construction both the owner-manager and the investor are ex ante indifferent between (R^1, R^2) and $(\widetilde{R}^1, \widetilde{R}^2)$, when holding $q^* = q^*_{old}$ constant. But as $q^*_{new} < q^*_{old}$, it follows from (A.5) ($d\Delta R^2/dq^* > 0$) that for the new optimal second-period contract, which implements some q^*_{new}, we have that $\Delta R^2(q^*_{new}, \widetilde{R}^1) < \Delta R^2(q^*_{old}, \widetilde{R}^1)$. Denote this contract by $\overline{R}^2$. Hence, $u_Y(\overline{R}^2, q) > u_Y(\widetilde{R}^2, q)$ holds for all q, and the ex ante expected payoff of the owner-manager with $(\widetilde{R}^1, \overline{R}^2)$ is strictly higher than with either $(\widetilde{R}^1, \widetilde{R}^2)$ or (R^1, R^2), respectively. To finish this step, note that by optimality of $\overline{R}^2$ the investor is also at least weakly better off with $(\widetilde{R}^1, \overline{R}^2)$ than with $(\widetilde{R}^1, \widetilde{R}^2)$, so that $(\widetilde{R}^1, \overline{R}^2)$ satisfies the investor's break-even condition. Taken together, this contradicts the optimality of R^1.

To conclude the proof, we can make use of the preceding results to show that, as asserted in the main text, in equilibrium the investor chooses a pure strategy and, thereby, implements the most efficient (i.e., lowest) q^* in case his optimal contractual choice at $t = 2$ is not uniquely determined. Given a debt security at $t = 1$, one can use the indifference condition (7) to express the second-stage levered equity security R^2 as a function of ΔR^1 and q^*. We can thus write $V(q^*, \Delta R^1)$ instead of $V(q^*, R^1)$ (cf. expression (A.8)). Further, we use $Q^* = \arg\max V(q^*, \Delta R^1)$ to denote the optimal choice correspondence subject to (12). Observe now that given R^1, $V(q^*, \Delta R^1)$ is strictly submodular in q^* and ΔR^1 :

$$\frac{\partial^2 V(q^*, \Delta R^1)}{\partial q^* \partial \Delta R^1} = -\frac{(p_{NB}p_{YG} - p_{YB}p_{NG})}{p_Y(q^*)^2} \int_{q^*}^{1} p_Y(q)\, dF(q) < 0.$$

Therefore, again by monotonic selection arguments, relaxing the investor's ex ante participation constraint by increasing ΔR^1 results in a lower set Q^*. Since Q^* is monotonic, it must be almost everywhere a singleton and continuous. Then, while the investor's payoff is continuous in ΔR^1 everywhere, the owner-manager's expected payoff is continuous a.e. and, where Q^* is not a singleton, the owner-manager strictly prefers the lowest (most efficient) value $q^* = \min Q^*$. Consequently, analogously to a tie-breaking condition, by optimality for the owner-manager the investor must choose $q^* = \min Q^*$ with probability one in equilibrium. **Q.E.D.**

Proof of Proposition 3. Recall from Proposition 1 that if the investor implements q_{FB}, then $u_N(R^1, q) = u_Y(\widehat{R}, q)$ holds for all $q \in [0, 1]$. Using this and the identity $s_d(q) = v_d(R^t, q) + u_d(R^t, q)$ to plug into (12), if the investor just breaks even at $t = 1$, one can express ΔR^1 as

$$\Delta R^1 = \Delta x - \frac{S_{FB} - (x_l - R_l^1)}{p_N(\widehat{q})}. \tag{A.10}$$

A first-period security that satisfies (A.10) is feasible if

$$\begin{aligned}
x_l &\geq R_l^1 \geq 0, \\
\Delta x &\geq \Delta R^1 = \Delta x - \frac{S_{FB} - (x_l - R_l^1)}{p_N(\widehat{q})} \geq 0, \\
R_l^1 &\geq \left(\frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{p_{YG} - p_{YB}} \right) \frac{S_{FB} - (x_l - R_l^1)}{p_N(\widehat{q})},
\end{aligned}$$

where the last inequality is just condition (9) from Proposition 1. These three conditions can be rewritten as follows:

$$\min\left(x_l, x_l + p_N(\widehat{q})\Delta x - S_{FB}\right) \geq R_l^1 \geq \max\left(x_l - S_{FB}, \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{NG} - p_{NB})p_Y(\widehat{q})} \left(S_{FB} - x_l\right) \right).$$

Since the left-hand side must be greater than the right-hand side, it must be that

$$x_l \geq \max\left(x_l - S_{FB}, \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{NG} - p_{NB})p_Y(\widehat{q})} \left(S_{FB} - x_l\right) \right) + \max\left(0, S_{FB} - p_N(\widehat{q})\Delta x\right).$$

Simple transformations yield condition (14). If (14) holds, by optimality for the owner-manager we then have that $q^* = q_{FB}$: The optimal security R^1 then maximizes joint surplus and, by making the investor just break even, achieves the maximum feasible payoff for the owner-manager.

We finally formalize the argument from the main text that in equilibrium $q^* > q_{FB}$ if (14) does not hold. That is, though first-best efficiency could be achieved by granting the investor a sufficiently large payoff, this is not optimal. Using the optimality of debt, consider the owner-manager's optimal choice of ΔR^1. Differentiating her expected profits with respect to ΔR^1 yields at points of differentiability of $q^*(\Delta R^1)$

$$\begin{aligned}
& \left(s_N\left(q^*\right) - s_Y\left(q^*\right)\right) f\left(q^*\right) \frac{dq^*}{d\Delta R^1} \\
& -\frac{d}{d\Delta R^1} \left(\int_0^{q^*\left(\Delta R^1\right)} v_N\left(R^1, q\right) dF\left(q\right) + \int_{q^*\left(\Delta R^1\right)}^1 v_Y\left(R^2, q\right) dF\left(q\right) \right) \\
& -\frac{d}{dq^*} \left(\int_0^{q^*\left(\Delta R^1\right)} v_N\left(R^1, q\right) dF\left(q\right) + \int_{q^*\left(\Delta R^1\right)}^1 v_Y\left(R^2, q\right) dF\left(q\right) \right) \frac{dq^*}{d\Delta R^1} \\
= & -\left(s_Y\left(q^*\right) - s_N\left(q^*\right)\right) f\left(q^*\right) \frac{dq^*}{d\Delta R^1} - \left(\widehat{p}_N F\left(q^*\right) + \frac{p_N\left(q^*\right)}{p_Y\left(q^*\right)} \widehat{p}_Y \left(1 - F\left(q^*\right)\right) \right),
\end{aligned}$$

where we used that the third line is zero by the investor's FOC at $t = 2$. Similarly, $dq^*/d\Delta R^1$ is computed from the solution to the investor's optimization problem at $t = 2$ and, from (7), we use that ΔR^2 is a function of ΔR^1 in the last line. This expression is strictly negative at $q^* = q_{FB}$, since then the first term is zero. **Q.E.D.**

Proof of Proposition 4. To obtain condition (17), note first that by construction $\Delta x \geq \Delta \widehat{R}^M \geq 0$ is always satisfied from feasibility of R^1 and from condition (1). Further, $\widehat{R}_l^M \geq 0$ never binds as after substitution

$$\widehat{R}_l^M = R_l^1 + \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{p_{YG} - p_{YB}} \Delta R^1 + K_2.$$

The remaining condition $x_l \geq \widehat{R}_l^M$ transforms to (17). Having derived $\widehat{R}^M$ this way, (1) implies that $\Delta \widehat{R}^M \leq \Delta R^1$ and $\widehat{R}_l^M > R_l^1$, where the inequalities are strict if $\Delta R^1 > 0$.

By the arguments in the main text, in equilibrium refinancing is obtained by types $q > q_{FB}$ but not by types $q < q_{FB}$, and types $q_{FB} < q < 1$ must obtain refinancing by issuing $\widehat{R}^M$. Further, the offer is accepted with probability one. It is straightforward to support this outcome as a Perfect Bayesian Equilibrium by adequately choosing out-of-equilibrium beliefs. **Q.E.D.**

Proof of Proposition 5. The proof follows from a series of results. Arguing that $v_Y(R^2, q)$ can only cross $v_N(R^1, q)$ from below, we first show that there is no equilibrium in which the highest type that issues a certain security extracts the whole surplus from refinancing. For this result (Claim 2) we make use of the following auxiliary result.

Claim 1. *If a security R^2 satisfying $v_N(R^1, 0) < v_Y(R^2, 0)$ and $v_N(R^1, 1) > v_Y(R^2, 1)$ is feasible, so that $v_Y(R^1, q)$* crosses $v_N(R^1, q)$ from above, *then also the first-best security $\widehat{R}^M$ is feasible.*

Proof. From the definition of $\widehat{R}^M$ we have $v_Y(\widehat{R}^M, 0) = v_N(R^1, 0) < v_Y(R^2, 0)$ and $v_Y(\widehat{R}^M, 1) = v_N(R^1, 1) > v_Y(R^2, 1)$, so it follows that $\Delta R^2 < \Delta \widehat{R}^M$ to make sure that the slope of $v_Y(R^2, q)$ is strictly smaller than that of $v_Y(\widehat{R}^M, q)$. But then $v_Y(\widehat{R}^M, 0) < v_Y(R^2, 0)$ implies that $R_l^2 > \widehat{R}_l^M$. By assumed feasibility of R^2, we therefore have that $x_l > \widehat{R}_l^M$, so that (17) holds strictly. Hence, if (17) does not hold, $v_Y(R^2, q)$ can only cross

$v_N(R^1, q)$ from below. **Q.E.D.**

Claim 2. *For any security issued in equilibrium, the highest type that issues this security extracts strictly less than the full surplus from refinancing.*

Proof. Observe first that if some type q^H has no incentive to mimic a higher type, the same holds strictly for all types $q < q^H$. Next, if some type $q \in [0, q_{FB}]$ extracts more than the full surplus, she must be pooling with some type $q > q_{FB}$. Suppose therefore that $q^H \in (q_{FB}, 1]$ is the highest type that issues some security $R^2(q^H)$ and that q^H extracts (weakly) more than the full surplus from refinancing, i.e. $v_N(R^1, q^H) \geq v_Y(R^2(q^H), q^H)$. From Claim 1 we know that for any feasible security $R^2(q^H)$, $v_Y(R^2(q^H), q)$ can only cross $v_N(R^1, q)$ from below. Hence, it must be that $v_N(R^1, q) > v_Y(R^2(q^H), q) \geq v_Y(R^2(q), q)$ for $q \in [q_{FB}, q^H)$, where the last inequality follows as by incentive compatibility: $u_Y(R^2(q^H), q) \leq u_Y(R^2(q), q)$ and where $R^2(q)$ is the equilibrium security issued by type q. Hence, in this candidate equilibrium *all* types $q \in [q_{FB}, q^H)$ would extract more than the full surplus. We have thus obtained a contradiction, as the investor is then always better off with his outside option rather than refinancing these types. Hence, it must be that $v_Y(R^2(q^H), q^H) > v_N(R^1, q^H)$. **Q.E.D.**

We now define more formally the refinement D1.[17] Let $U(\widetilde{R}^2, q, \pi)$ be the expected payoff of the owner-manager when offering a security $\widetilde{R}^2$

$$U(\widetilde{R}^2, q, \pi) := \pi u_Y(\widetilde{R}^2, q) + (1-\pi)\, u_N\left(R^1, q\right).$$

For each type q, determine the minimum probability of acceptance, $\Pi(q|\widetilde{R}^2)$, that would make offering $\widetilde{R}^2$ weakly attractive

$$\Pi(q|\widetilde{R}^2) = \min\{\pi : U(\widetilde{R}^2, q, \pi) \geq U^*(q)\},$$

where $U^*(q)$ denotes the equilibrium payoff of type q. Then, provided that this leads to a non-empty set, D1 restricts the support of the investor's beliefs to those types that would find $\widetilde{R}^2$ attractive for the lowest probability of acceptance

$$Q^{dev}(\widetilde{R}^2) = \left\{ q \in [0,1] \mid \Pi(q|\widetilde{R}^2) = \min_{q'} \Pi(q'|\widetilde{R}^2) \right\}.$$

[17] Originally, as discussed in Cho and Kreps (1987), D1 was defined for discrete type spaces. The extension to continuous types follows, e.g., Ramey (1996) or DeMarzo et al. (2005).

Claim 3. *In an equilibrium satisfying D1, all types that obtain refinancing offer the same debt security.*

Proof. This follows standard arguments (cf. Nachman and Noe, 1994), so we omit the formal details of the proof for the sake of brevity. We showed that when the first best is not feasible, as (17) does not hold, then the highest type issuing a certain security, i.e., the highest type in the respective "pool", never extracts the full surplus. This type would thus strictly benefit from "separating away" from the pool. Given that higher types strictly prefer to share cash flow for the (less likely) low realization, this is possible under D1, provided that the initial security was not debt. Clearly, it is not incentive compatible to have more than one debt security in equilibrium. Finally, pooling with debt for all types who receive refinancing can be supported by beliefs that satisfy D1. **Q.E.D.**

Proof of Proposition 6. Suppose first that the investor just breaks even ex-ante, so that

$$\begin{aligned} \Delta R^1 &= \frac{K_1 - R_l^1}{p_N(\widehat{q})}, \\ \Delta R^2 &= \Delta x - \frac{R_l^2 - R_l^1 + p_N(q_M^*)\left(\Delta x - \Delta R^1\right)}{p_Y(q_M^*)}. \end{aligned} \tag{A.11}$$

(Recall that $\widehat{q}$ is the unconditional expectation of q.) Note that $R_l^2 = x_l$, so that we can represent the equilibrium security R^2 as a function of R^1 and q_M^* only. By plugging (A.11) into (19), one can express the binding ex ante participation constraint of the investor entirely as a function of R_l^1 and q_M^*

$$\begin{aligned} K_1 &= \int_0^{q_M^*} \left(R_l^1 + p_N(q)\frac{K_1 - R_l^1}{p_N(\widehat{q})} \right) dF(q) \\ &+ \int_{q_M^*}^1 \left(R_l^2 + p_Y(q) \left(\Delta x - \frac{x_l - R_l^1 + p_N(q_M^*)\left(\Delta x - \frac{K_1 - R_l^1}{p_N(\widehat{q})}\right)}{p_Y(q_M^*)} \right) - K_2 \right) dF(q). \end{aligned} \tag{A.12}$$

Taking the total derivative of (A.12) allows us, therefore, to examine how a change in R_l^1 affects the equilibrium cutoff q_M^* at the interim stage, given that R^1 and R^2 adjust so that the investor has the same ex ante expected payoff under the old and the new equilibrium.

From total differentiation we obtain

$$0 = \left[\begin{array}{c} \left(R_l^1 + p_N(q_M^*)\Delta R^1 - x_l - p_Y(q_M^*)\Delta R^2\right) f\left(q_M^*\right) \\ + \int_{q_M^*}^1 p_Y(q) \frac{d\Delta R^2}{dq_M^*} dF\left(q\right) \end{array} \right] dq_M^* \qquad \text{(A.13)}$$

$$+ \left[\int_0^{q_M^*} \left(1 - \frac{p_N(q)}{p_N(\widehat{q})}\right) dF\left(q\right) + \int_{q_M^*}^1 \frac{p_Y(q)}{p_Y(q_M^*)} \left(1 - \frac{p_N(q_M^*)}{p_N(\widehat{q})}\right) dF\left(q\right) \right] dR_l^1,$$

where for ease of exposition only we have plugged back in for ΔR^t in the first line. With overinvestment, $q_M^* < q_{FB}$, the first term in the first line is positive. Also the second term is positive, as $d\Delta R^2/dq_M^* > 0$.[18] Finally, the second line is also positive. To see this, note that differentiating the terms in front of dR_l^1 with respect to q_M^* we have

$$\int_{q_M^*}^1 \left[\frac{p_Y\left(q\right)}{p_N(\widehat{q})} \left(\frac{(p_{YG}p_{NB} - p_{NG}p_{YB}) - (p_{YG} - p_{YB})\, p_N(\widehat{q})}{p_Y(q_M^*)^2} \right) \right] dF\left(q\right) < 0.$$

Further, these terms are zero at $q_M^* = 1$, while $q_M^* \leq q_{FB} < 1$. Taken together, from the preceding observations on (A.13) we obtain $dq_M^*/dR_l^1 < 0$. As the owner-manager is the residual claimant and as $q_M^* < q_{FB}$, we thus have that R_l^1 is optimally chosen as small as possible: $R_l^1 = 0$.

It remains to show that it is optimal for the owner-manager to offer the investor a contract for which he *just* breaks even at $t = 1$. For this it is sufficient to show that q_M^* decreases (i.e. becomes more inefficient) as the investor's ex-ante payoff increases. To avoid new notation, note that we can likewise analyze a change in K_1, while still assuming that the investor just breaks even. Total differentiation yields then

$$0 = \left[\left(p_N(q_M^*)\Delta R^1 - x_l - p_Y(q_M^*)\Delta R^2\right) f\left(q_M^*\right) + \int_{q_M^*}^1 p_Y(q) \frac{d\Delta R^2}{dq_M^*} dF\left(q\right) \right] dq_M^*$$

$$+ \left[\int_0^{q_M^*} \frac{p_N(q)}{p_N(\widehat{q})} dF\left(q\right) + \int_{q_M^*}^1 \frac{p_Y(q)}{p_Y(q_M^*)} \frac{p_N(q_M^*)}{p_N(\widehat{q})} dF\left(q\right) - 1 \right] dK_1.$$

Since the terms in the second line are positive, it must be that $dq_M^*/dK_1 < 0$.[19] **Q.E.D.**

Proof of Proposition 7. We only have to check the feasibility requirements for R^1 and

[18] See (A.11) and (A.5) and recall that $R_l^2 = x_l$.

[19] Similarly to above, to verify this note that differentiating the second line with respect to q_M^* yields

$$-\int_{q_M^*}^1 \left[\frac{p_Y(q)}{p_N(\widehat{q})} \frac{(p_{YG}p_{NB} - p_{NG}p_{YB})}{p_Y(q_M^*)^2} \right] dF\left(q\right) < 0.$$

$\widehat{R}^M$:

$$\begin{aligned}
x_l &\geq R_l^1 \geq 0, \\
\Delta x &\geq \Delta R^1 = \frac{K_1 - R_l^1}{p_N(\widehat{q})} \geq 0, \\
x_l &\geq \widehat{R}_l^M = R_l^1 + \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{p_{YG} - p_{YB}} \frac{K_1 - R_l^1}{p_N(\widehat{q})} + K_2,
\end{aligned}$$

where the last condition is just (17) from Proposition 4. These conditions can be rewritten as

$$\begin{aligned}
R_l^1 &\leq \min\left(\frac{(p_{YG} - p_{YB})\, p_N(\widehat{q})}{(p_{NG} - p_{NB})\, p_Y(\widehat{q})} \left(x_l - K_2 - \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{YG} - p_{YB})\, p_N(\widehat{q})} K_1 \right), x_l \right), \qquad \text{(A.14)} \\
R_l^1 &\geq \max\left(0, K_1 - p_N(\widehat{q})\Delta x\right),
\end{aligned}$$

where we have already used that $x_l < K_1$. One can construct a feasible security R^1 only if the right-hand side in the first line is greater than the right-hand side in the second line. Note now that from $x_l < K_1$ the first term on the right-hand side of (A.14) is the smallest. We then have the requirement

$$\begin{aligned}
& \frac{(p_{YG} - p_{YB})\, p_N(\widehat{q})}{(p_{NG} - p_{NB})\, p_Y(\widehat{q})} \left(x_l - K_2 - \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{YG} - p_{YB})\, p_N(\widehat{q})} K_1 \right) - x_l \\
= \ & \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{NG} - p_{NB})\, p_Y(\widehat{q})} (x_l - K_1) - \frac{(p_{YG} - p_{YB})\, p_N(\widehat{q})}{(p_{NG} - p_{NB})\, p_Y(\widehat{q})} K_2 < 0,
\end{aligned}$$

which after some transformations becomes condition (20) in the main text. **Q.E.D.**